A DUALISTIC MODEL OF ULTIMATE REALITY AND MEANING: SELF-SIMILARITY IN CHAOTIC DYNAMICS AND SWEDENBORG*

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INTRODUCTION

In the effort to find meaning and reality we explore diverse realms of knowledge. Two such realms, theology and science, share a mutual history of divergence and even conflict. One of the better known historical examples involves the case of Galileo Galilei (1564–1642) who confronted the Catholic hierarchy with the view that the earth orbited the sun—a position that countered the contemporary dogma of a geocentric solar system. While Galileo recanted his view and the church position prevailed for the moment, Galileo's ideas ultimately triumphed. Galileo's case is indicative of the pattern of slow but steady assumption by science of the right to be the sole interpreter of the natural world. Before this onslaught of scientific data and logic, theology has retreated to the point where Richard Altick could make the observation, even at the end of the 19th century, that science "finally made unbelief respectable" (Altick, 1973, p. 233). Current scholarship involving the interface of theology and science—where it is found—often continues the confrontational trend with the elucidation of contradictions between biblical references and current scientific theories of, for example, evolution and geological morphogenesis (see, for example, Clements, 1990). But even more telling is the fact that, in the much larger world of science, scientific reporting and discussion as found in current scientific journals completely ignores the religious dimension. Scientific journals are extremely reluctant to consider publica-

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tion of manuscripts that do contain theological discussion. And insofar as the purpose of such journals is to report science, this exclusion seems justified.

Yet there are thoughtful scientists who have religious convictions and perceive the need of a religious connection with their scientific views (Baker, 1992). Perhaps the time is past when theology can prescribe an explanation of natural reality, but for the religiously-minded there should be other, perhaps more subtle, kinds of connectivity.

In this paper I will propose a possible general connection between science and religion—a *correspondence relation.* While a correspondence relation can occur in a multitude of ways, we will discuss a specific example involving an approximate isomorphism between some structures in chaotic dynamics and a specific theological structure or paradigm of God, humankind, and creation. Therefore we begin in the following section, entitled "Chaotic Dynamics," by describing certain features of the new science of unstable deterministic systems called *Chaos.*

Chaotic phenomena are found in biology, chemistry, physics, and mathematics as well as the applied fields of medicine and engineering. From the fibrillation of the heart, to unstable chemical reactions, to the rings of Saturn, chaotic behavior is ubiquitous. For science, it is a new realization of what has always been manifest, that nature really is complex and marvelous.

Chaotic systems are characterized by several mathematical properties, including the nonlinearity of the appropriate equations and an unusual *fractal* geometry. In this paper we focus on the fractal geometry property as the scientific member of the corresponding religion / science pair. Fractals exhibit two properties—variety and self-similarity—that will form the basis of the correspondence relation with theology. The specific theology of this proposed isomorphism is that reported by the 18th century philosopher and revelator Emanuel Swedenborg. Therefore, in the second major section of the paper we attempt to show that Swedenborgian theology also exhibits the characteristics of self-similarity and variety. We hope that this alignment of these two structures will provide a plausible demonstration of the validity of the correspondential connection between religion and science.

CHAOTIC DYNAMICS

Fractals—the Geometry of Chaos

In his book *The Geometry of Fractals* Benoit Mandelbrot (1983)—a founder of the subject—suggests a first example of a fractal that is drawn from nature rather than mathematics—the coastline of an island. Because of its typically jagged shape, the coastline has some remarkable properties. Consider the problem of measuring its length. If one uses a fairly coarse measuring device, the length is readily measurable and finite. But as the measuring device becomes capable of discerning smaller distances and curvatures, one finds the coastline to be longer; and as the measuring device becomes infinitely precise, the coastline—in theory—becomes infinitely long! Yet, the contained land-mass is still finite. Furthermore, the coastline has the property that, if a short section is enlarged, it appears similar to an unmagnified long section. Hence the patterns are repeated on different scales. Mathematicians call this phenomenon self-similarity.

To begin our analysis of fractals (Barcellos, 1984), let us consider a very simple fractal known as the Cantor set, named for its originator, the mathematician Georg Cantor. The Cantor set is formed by repeated operations on the points of a real number line. First start with a line segment extending from zero to one. Then remove the middle third, leaving two pieces at either end. Repeat the process an infinite number of times on each of the pieces produced at each iteration. As the number of iterations tend to infinity the resulting set is the Cantor set.

Clearly, the Cantor set has some unusual properties. First, the generating process is itself fairly novel. Second, the set has an infinite number of points but, as can be shown mathematically, has a length of zero. Third, and running counter to our intuition about geometry, the Cantor set has a fractal dimension, lying between the dimension of one (for a finite line segment) and a dimension of zero (suitable to a finite collection of points). Expressed mathematically its dimension is:

$$\log 2/\log 3 = 0.6309298 \ldots$$

Thus, the Cantor set seems to exist in some sort of limbo between points and lines.

While we are familiar with integer values of dimension—zero (0) for a point, one (1) for a line, two (2) for a plane, and so on—the concept of dimension can be generalized to include the notion of scaling, or the behavior of a geometric configuration in the large and in the small. Dimension provides a measure of the object as one scales to smaller sizes.

To make this idea more concrete let us first apply it to a line, then to an area, and finally to the fractal Cantor set. (See Figure 1.) We will see that scaling provides the "right" answer for the line and plane, and then gives the fractional dimension for the Cantor set.

Consider first the line of unit length in Figure 1. We subdivide the line repeatedly and count the number of "boxes" required to "cover" the line at each repetition. Of course, the box size shrinks by a compensating factor at each iteration. The tables given in the figure illustrate the process for the line and plane. In each case relations between N and ε are expressed as equations, all similar in form except that the exponent on the right side of each equation is different for each configuration. This exponent provides the invariant quantity we call dimension. The values of 1 and 2 for the line and plane are intuitively correct. While not as obvious, the fractional value for the Cantor set is derived in a manner consistent with this approach as shown in Figure 2. In this way dimension is an invariant characteristic in the scaling of the configurations to very small sizes. And this invariance is the property known as *self-similarity.*

Fractals can be generated in a variety of ways. But our purpose here is to illustrate their appearance in science, as nature would manifest them. Our particular manifestation is that found in chaotic dynamics, and therefore we first describe the general features of dynamical systems and then the special nature of chaotic dynamical systems. Finally we illustrate the various characteristics of chaotic dynamics as they appear in a typical dynamical system—the driven pendulum.

Dynamical Systems

Dynamics is the study of the relation between force and motion, and in physical science dynamics utilizes mathematics to describe and deterministically predict states of a physical system. One of the earliest such

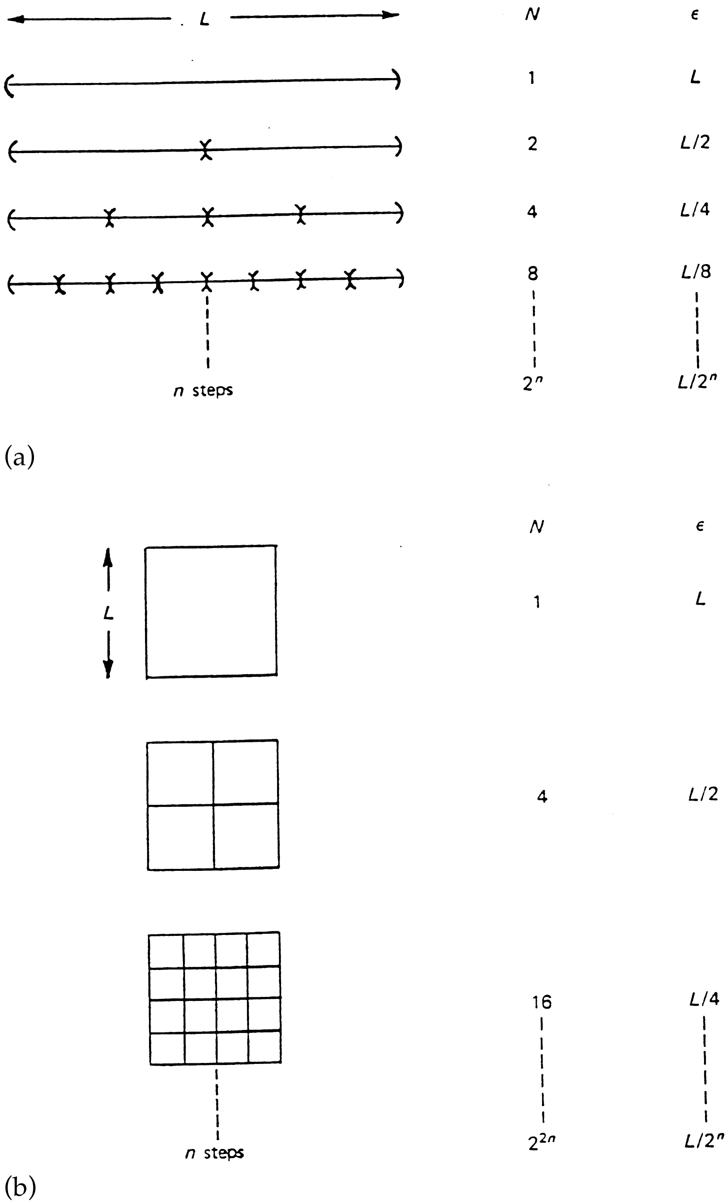
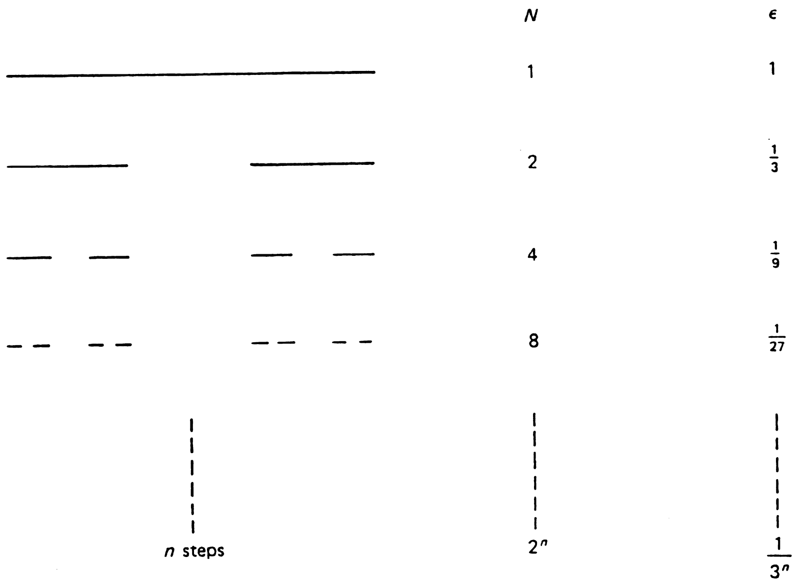


Fig. 1. The box covering method is used to calculate dimension. Increasingly smaller boxes are used to "cover" the configurations. "Boxes" cover (a) first a line and then (b) an area.



$$d = \lim_{\epsilon \to 0} \left(\frac{\log N}{\log (1/\epsilon)} \right) = \lim_{n \to \infty} \left(\frac{\log 2^n}{\log 3^n} \right) = \frac{\log 2}{\log 3} < 1$$

Fig. 2. The Method illustrated in Fig. 1 is applied to a Cantor set, leading to a non-integer value of dimension.

mathematical relations is a differential equation that describes Newton's second law—mass times acceleration equals force:

$$m \frac{dv}{dt} = F$$

More recently, *any* system specified by a set of differential equations (or difference equations) is a *dynamical system.* Therefore we can define a dynamical system, somewhat abstractly, as a set of equations that characterizes the time-dependent behavior of certain quantities. Examples of these quantities include the concentrations in a chemical reaction; the charge, current, and voltage in an electrical circuit; or the angular velocity and displacement of a driven pendulum. In these particular examples, the time-dependent behavior may be modeled by chemical kinetic theory, Ohm's electrical law, and Newton's second law, respectively. But whatever the particular example or corresponding governing law, the study of dynamical systems attempts to find the time-dependent behavior of the given quantities, known generally as *dynamical variables.*

Until the advent of computers, dynamical systems were usually solved by mathematical analysis, *where possible.* Where analytic solutions could *not* be found, the system, or at least that version of the system, was often relegated to the backwaters of science. Systems that did provide analytic solutions usually exhibited stable, regular (often periodic) motion. One large class of such systems is that of linear systems, and their ubiquity in science provides a *raison d'être* for the emphasis in scientific education on linear algebra, linear differential equations, and linear systems analysis. The existence of this variety of analytic tools has given linear models of nature a much-favored status in science over more general dynamical systems.

Yet dynamical systems can have unstable motions when the dynamical variables are coupled in a *nonlinear* fashion. This circumstance allows for a delicate and complex interplay of motions. While nonlinear systems probably represent a more accurate modeling of many physical processes than that provided by linear models, such systems are, as we have suggested, difficult to analyze. Early attempts by the French mathematician Henri Poincare form the basis for current analytic studies in nonlinear systems (Guckenheimer and Holmes, 1983). Yet the primary catalyst for the study of these complex systems has been the advent of digital computers. Computers can provide numerical solutions where analytic solutions are very difficult or impossible to find. This numerical work does, in turn, promote the development of new mathematics and new insight into the nature of complexity.

Chaotic Dynamical Systems

Chaotic dynamical systems are therefore a subset of dynamical systems. Before discussion of a particular system, let us describe the general characteristics of chaotic dynamics. First, because chaotic systems are a subset of dynamical systems they are *deterministic.* (In this discussion we ignore the somewhat less defined area of quantum chaos). The deterministic property implies that there is some underlying law, often expressible as a differential equation, that describes the time evolution of the system.

Chaotic systems must be *nonlinear* since linear systems always have periodic or regular motion. Furthermore, the chaotic system needs some complexity; there must be a sufficient number of *degrees of freedom* so that the system can exhibit the complicated motions of chaos.

Finally chaotic dynamics usually results from some sort of competition within the dynamical system—often between the *natural* motion of the system and a *forcing* motion that is imposed upon the system. The complex interplay between these motions can result in a variety of effects. Sometimes one motion is dominant over the other, and the system is said to "lock" onto a particular motion. At other times neither motion dominates, and the dynamics are completely unstable and chaotic.

These characteristics of chaotic dynamics lead to a variety of consequences. A very prominent feature is the property of *sensitivity to initial conditions* (SIC). We recall that chaotic systems are deterministic and therefore if a particular system is given certain initial conditions—initial position, initial velocity, and so on—then its trajectory or path is completely defined by the equations governing the dynamical system. Its motion will always be specified. Yet, in the real physical world, this kind of precision in the specification of the initial state is typically unavailable. Furthermore, computations of physical quantities involve round-off errors. Even the largest computers introduce such errors that lead to further computational uncertainty (Fryska and Zohdy, 1992). In most cases there are only estimates of initial conditions. For nonchaotic dynamical systems such vagueness does not matter in that trajectories that start fairly close to each other tend to stay that way. For example, two coins dropped at approximately the same time from a tall building will tend to land at roughly the same time. But for chaotic systems, discrepancies in starting points can lead to widely varying trajectories; hence the expression "sensitivity to initial conditions."

The SIC property leads to consequences in regard to the correlation between two initially close systems. Trajectories that start close together soon diverge from each other and, in technical terms, we speak of a *short correlation time* for the system. In general, motion can be characterized by its correlation time. If motion has a very long correlation time, then it is completely regular (non-chaotic). If there is no correlation between initially close trajectories, then the system is *random* or *stochastic*—random in time. The lack of regular or periodic motion in a chaotic system leads to an *appearance* of randomness. If the system were truly random, the methods of statistical mechanics could be used for analysis. Yet chaotic systems are fundamentally deterministic and, while the data may have a "randomness" quality, the dynamics are fundamentally different. Clearly, in the spectrum of possible motions, chaotic dynamics sits somewhere between regular and random motion. For example, this spectrum can be represented by the varieties of fluid flow. Smooth laminar fluid flow is regular whereas fully developed turbulence is random. Chaotic dynamics is found in the intermediate region at the onset of turbulence. Those who are interested in efficient aircraft or quiet submarines have a keen interest in the chaos of fluids.

The finite correlation time of chaotic motion also leads to the notion of finite prediction time. No real system is specified with infinite precision, and therefore the sensitive dependence on initial conditions causes an initial uncertainty to build rather quickly to encompass the whole range of possible values of the dynamical variables. Once the uncertainty has built to occupy all possibilities, the notion of prediction is completely lost.

As we shall see, the dynamics of chaos can be represented pictorially in a geometric "space" of the dynamical variables of the system. In this space the geometry is fractal, and the dimensionality of the resultant structures is typically fractional. The study of dimensionality and correlation especially marks chaotic behavior as a new and unique dynamical state. With these tools, chaotic dynamics can often distinguish randomness and determinism in experimental data. Such distinctions have been made in a variety of arenas including physics, chemistry, biology and medicine (see, for example, Theiler et al, 1992).

We now attempt to make these ideas more concrete through the use of a specific dynamical system—the driven pendulum. With this example system we illustrate the primary concepts of chaotic dynamics and lead the discussion toward the very important connection of chaotic dynamics with the geometry of fractals. Ultimately we wish to demonstrate that chaotic dynamics provides an important model for self-similarity.

The Driven Pendulum

The pendulum has long been an object of scientific enquiry—at least since Galileo's observations of swinging lamps in the cathedral at Pisa toward the end of the 16th century. The *driven* pendulum actually has much in common with the motion of a child's pushed swing. Typically the swing is pushed at the so-called *resonant* frequency to provide maximum energy transfer. But suppose that it were pushed at a different frequency. Experience shows that the swing then oscillates with a smaller amplitude and the child receives a bumpy ride. However, if the swing were pushed *very* hard at a non-resonant frequency, it would execute complex motions including looping over the top of the swing axis—an obviously unsafe ride for the child! As the amplitude is increased the motion goes through alternate regimes of periodic and unstable, chaotic behavior.

The motion of the driven pendulum (Baker and Gollub, 1990) is governed by Newton's second law. The forces acting on the pendulum are (a) the frictional damping of motion; (b) the restoring force of gravity that pulls the pendulum bob toward its rest position; and (c) a periodic driving or pushing force that continually energizes the system. Manipulation of Newton's law provides a mathematical form which is manifestly that of a dynamical system:

$$d\omega/dt = -(1/q)\omega - \sin\theta + g\cos\phi,$$
$$d\theta/dt = \omega,$$
$$d\phi/dt = \omega_{D.}$$

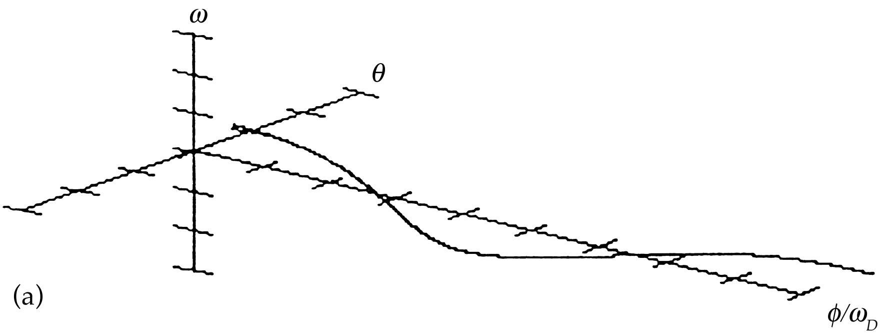
The presence of the nonlinear $\sin\theta$ terms and the driving force, $g \cos \phi$, are necessary for chaotic behavior. The three dynamical variables are θ, the angular displacement of the pendulum, ω, the angular velocity of the pendulum bob, and, ϕ, a variable proportional to the time. This system is periodic (stable) or chaotic (unstable) depending on the values of three parameters, q, the damping factor, g, the amplitude of the driving force, and, ω_D, the angular frequency of the periodic driving force. Each parameter plays a different role. Increased damping tends to stabilize the motion, whereas increased forcing tends to destabilize the motion. Values of the drive frequency ω_D that differ from the natural frequency of unity, can precipitate a destabilizing competition between the force of gravity and the driving force. Because of all these factors, the pendulum is capable of exhibiting a variety of periodic and chaotic motions.

The fractal nature of chaotic pendulum motion is evident in certain geometric constructions involving the dynamical variables (ω, θ, ϕ). An understanding of these constructions requires a little background knowledge. We begin with a description of 'phase' space. The phase space of a dynamical system of N variables is an N-dimensional Euclidean space, with each axis corresponding to one of the variables. For the pendulum, the axes are labeled as ω, θ, and ϕ. The real space motion—as we see it—is illustrated quite differently in phase space by a sequence of points moving positively in the ϕ (or time) direction, with ω and θ taking their instantaneous values in the plane perpendicular to the axis.

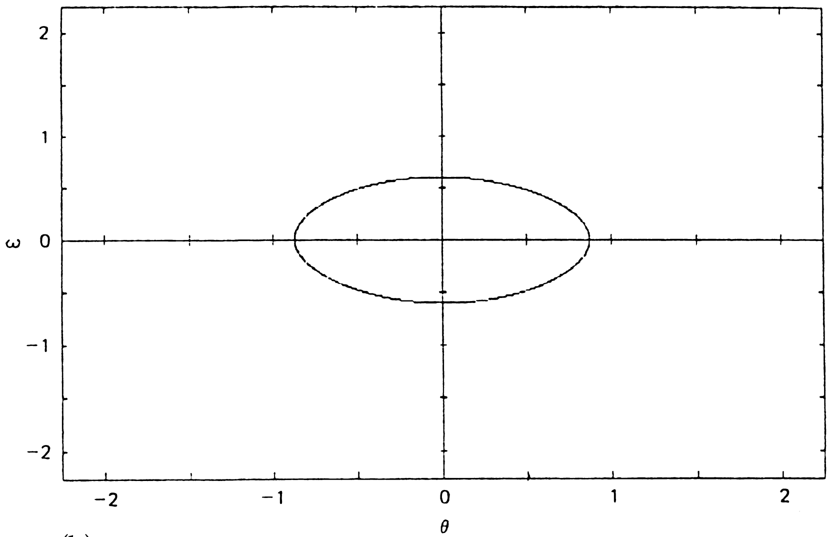
For example, if the motion in real space is a periodic, back and forth motion, then, when the angular displacement θ is greatest, the angular velocity ω is zero. Similarly, when the pendulum is at the bottom of its motion, θ is zero and ω has its largest magnitude. Therefore the motion in phase space is periodic; a spiral directed positively along the time axis. (The conditions on the time axis are such that when the phase orbit reaches the end of the axis, it automatically goes back to the beginning of the axis. Therefore the single spiral is actually a superposition of the many identical spirals occurring over a significant time period. In technical language the cyclical nature of of the time coordinate is a *periodic boundary condition*). See Figure 3(a). Figure 3(b) is a two dimensional phase space representation where the time dimension is compressed into the plane.

An important aspect of this diagram is that it also represents the *attractor* of the motion: the set of points to which the phase-space motion is eventually drawn. The spiral is the phase-space motion after any initial transient behavior has died away. Whatever the initial trajectory of the pendulum, its phase space motion is "attracted" to the spiral and therefore the spiral is the attractor.

Now consider the motion of a pendulum that is pushed vigorously at a frequency that differs from its natural frequency. The motion may be



(a)



(b)

Fig. 3. Geometrical representations of the dynamics of a simple oscillating pendulum. In (a) the angle, angular velocity, and time form a triplet of axes for the motion. The back and forth motion is represented in this new "space" by a spiral. In (b) the time dimension is suppressed and the motion in the two dimensional space of angle and angular velocity forms an ellipse.

quite irregular (and for the child on a comparable swing, very unpleasant). The real space motion of the pendulum is unstable, varying continuously with no perceivable pattern and quite sensitive to starting values of the variables. Even after the initial transients have died away, the motion is still very complex and always changing. In phase space the attractor is now correspondingly complex, with an apparently infinite number of foldings. A motion picture simulation of the attractor along the time axis would show a continual stretching and folding as nearby phase points diverge from each other. See Figure 4(a) for the full three dimensional phase diagram. Figure 4(b) shows a two dimensional representation by compressing the time axis.

Because chaotic attractors are so very complex, it is sometimes convenient to use only a two-dimensional cross section of the attractor that is cut across the ϕ axis. The section is a plane, with ω and θ as the coordinate axes. This device, called a *Poincaré section,* is especially useful when the systern is driven periodically, since it always has the same appearance at a particular point in the drive cycle.

Perhaps the most striking feature of the chaotic Poincare attractor is that, at any point on the attractor, there is a local direction along which the attractor seems to flow quasi-continuously, and there is a corresponding perpendicular direction along which the attactor has a fractal appearance like that of a Cantor set. Therefore the attractor displays self-similarity. In general, damped chaotic systems do have fractal attractors, and for this reason these special attractors are called *strange attractors.*

Besides the self-similarity property, the strange attractor also exhibits a fascinating structural variety beyond that suggested by the Cantor set appearance. While the gross structure is Cantor-like, the details vary endlessly throughout the attractor and at different levels of magnification. In fact it is generally true that chaotic attractors exhibit both self-similarity and variety.

(Naturally occuring fractals also exhibit both self-similarity and variety. Typical examples would include dendrites on crystals and the vascular system of the human body.)

As a further confirmation of the fractal nature of the attractor one can calculate (with a computer) the fractal dimension of the Poincare attractor and of the full attractor embedded in the three-dimensional space. In this state—as characterized by a particular set of parameters—the Poincare section has a dimension of 1.38 and the full attractor has a dimension of 2.38. As expected, the dimensions are non-integer. Of course in the non-

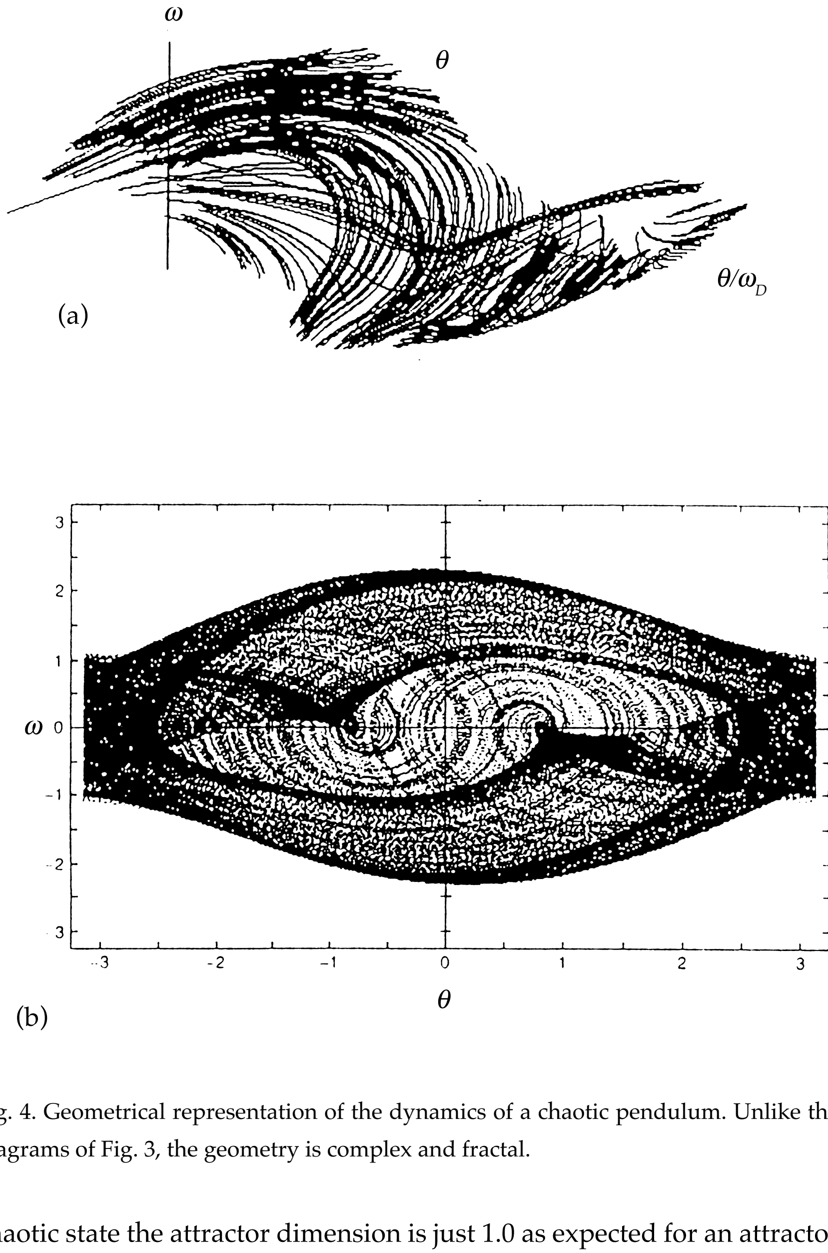


Fig. 4. Geometrical representation of the dynamics of a chaotic pendulum. Unlike the diagrams of Fig. 3, the geometry is complex and fractal.

chaotic state the attractor dimension is just 1.0 as expected for an attractor that is a simple curve. Figure 5(a, b, and c) show the Poincare section at increasing magnification. One feature of such chaotic attractors is that the

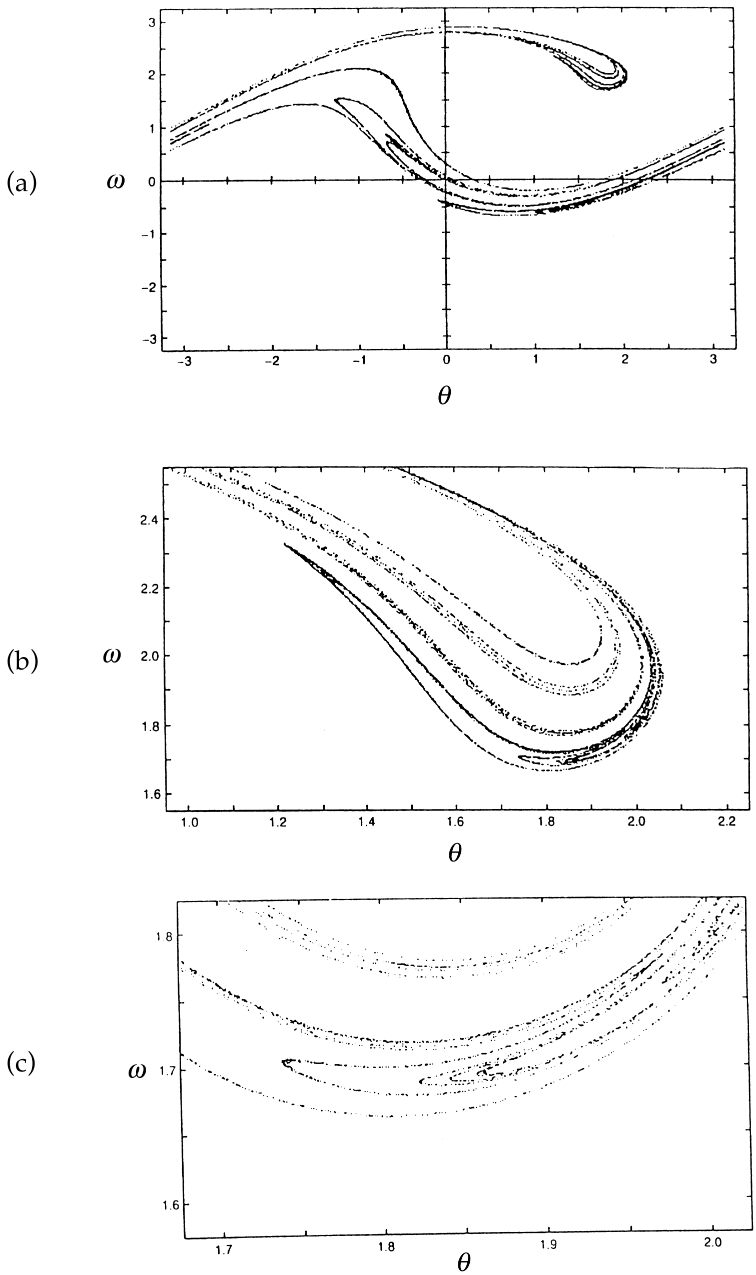


Fig. 5. Increasingly magnified views of the Poincaré section illustrate the concept of approximate self-similarity.

more an object is magnified, the more the internal structure becomes apparent. Furthermore, and even more marvelous, is the fact that the structure seen under magnification more or less resembles the gross structure. This is an illustration of the self-similarity property described earlier.

We recall that a main characteristic of chaotic dynamics is sensitivity to initial conditions. The block of states stretches significantly but, because of the dissipative nature of the system, the area of the block or the number of dynamcal states actually shrinks. As time goes on the "shape" of the states will spread out as a fine filament and cover much of the available phase space. Hence the system becomes indeterminate and prediction of future states is impossible. One sees a progressive loss of information about the system. The rate of loss is a property of the dynamics and is, in fact, related to the fractal dimension of the strange attractor. Furthermore, it seems likely that this loss of memory in chaotic systems goes to the heart of the nature of irreversible processes (Mackey, 1991). Clearly these features all suggest deep connections between geometry, dynamics, and even the directionality of time itself.

SPIRITUAL SELF-SIMILARITY IN THE PHILOSOPHY OF EMANUEL SWEDENBORG

Our central themes of self-similarity and variety are also found in the theology described by the eighteenth century scientist, philosopher, and revelator Emanuel Swedenborg. The ubiquity of this framework in his writing demonstrates the self-similar nature of Swedenborg's philosophy. We present two examples.

Universal Dualistic Structure

Swedenborg characterizes the state of something as having two aspects; *esse* (being) and *existere* (taking form) (Swedenborg, 1763, § 14). A comparable pairing would be *substance* and *form.* This characterization we refer to as the *universal dualistic structure* because it is ubiquitous in the Swedenborgian worldview. For example, God is described in these terms and all is created by God and from God. God consists of infinite or divine love and infinite or divine wisdom. Love is the essence of God and wisdom is the form that the love takes. Without wisdom as its form, the love is shapeless and impotent. Wisdom gives structure and order to the manifestation of God's love in creation and in the relationships between God and humankind.

Creation is described as an actualization of God's infinite dualistic structure. And the most complex part of creation is the human being because the human being is a reflection of God. Hence humans are in the image of God, and therefore humans also possess the dualistic structure. The basis of humanity lies in the love (being) and wisdom (form of love) of people; qualities that are paralleled by the human capacities to act freely (from the will), and act rationally (from the understanding). Therefore human love and wisdom, will and understanding, and freedom and rationality, are all different shades of the universal dualistic structure.

This duality is also reflected in gender differences. Men and women complement each other and, in fact, were "... created to be the very form of the marriage of good and truth" (Swedenborg, 1768, § 100). In Swedenborg's words, "... the male was created to be the understanding of truth ... and the female was created to be the will of good ..." (Swedenborg, 1768, § 180). The most profound human relationship is therefore the conjunction of man and woman through an eternal marriage leading to a state that Swedenborg describes as one of "... innocence, peace, tranquillity, inmost friendship, full confidence, and a mutual desire of mind and heart to do each other every good; and from all these come blessedness, happiness, joy, pleasure, ... and heavenly happiness" (Swedenborg, 1768, § 180). Furthermore, this relationship between a man and woman also pictures the ideal relationship between God and the church—a collective human response to God (Swedenborg, 1768, § 116).

Swedenborg relies heavily on the notion of *correspondence* to connect various aspects of creation and to describe spiritual qualities in terms of material objects. A correspondence relation is taken to be a parallelism between function and generalized form. Usually it involves a spiritual-natural pairing. As a theoretical biologist, Swedenborg often utilized a correspondence of the heart with the will or good, and the lungs with the understanding or truth (Swedenborg, 1763, § 378). Swedenborg used the biological functions of these organs to illustrate spiritual processes. In a similar spirit we might now wonder if the dual molecular bases of life, protein and DNA molecules, or even the approximate reflection symmetry of the body might also be examples of this universal dual structure.

Even Swedenborg's description of the afterlife is couched in terms of a dualistic heaven—a heaven that is partitioned according to the nature of its inhabitants. There is a so-called *celestial* heaven in which the inhabitants are somewhat child-like and have an overwhelming love of God and secondarily a love of their fellow beings, and there is a so-called *spiritual* heaven in which the inhabitants are less trusting, more rationalistic, and focus on love of their neighbor first and love of God secondarily. Inhabitants of the celestial heaven are more affective (of the will) and those of the spiritual heaven are predominantly cognitive (of the understanding) (Swedenborg, 1758, § 20). Again one sees, the dualistic structure. Swedenborg uses the dualistic model throughout his descriptions of spiritual phenomena.

Greatest and Least

In the discussion of the Universal Dualistic Principle the emphasis was on the dualism and many structural examples that follow from it. Thus the dualism is universal. Another facet of this universality is the Swedenborgian statement found in *Divine Love and Wisdom* that the "The Divine is the same in the greatest and the smallest things" (Swedenborg, 1763, § 77). The terms "greatest and least" are used in a very broad sense. Swedenborg refers to God being in the entire Church or the entire Heaven, and yet being the same in a single person or angel. Or in a different sense of "greatest and least," he says that the Divine is the same in an elderly person and an infant. Of course, the degree of reception may be different from one entity to another, yet in the statement it is emphasized that God is the same in all. For inanimate objects Swedenborg says, "The Divine is also the same in the greatest and smallest parts of all created things which are not alive" (ibid, § 80). The "greatest and least" idea is therefore even more general than the universal dualistic principle, in that it suggests that at every level of creation and in every process of creation, there is some sort of correspondential self-similarity. And of course there is always variety.

Spiritual phenomena and religious life can provide many examples of self-similarity and variety within self-similarity. And these examples from religion, have some sort of correspondence with the more obvious self-similarity and variety exhibited by the fractals of chaotic dynamics. In this case, religion and science meet in a correspondence relation. But perhaps this connection through self-similarity and variety serves a deeper principle of reality. The following quotation from Swedenborg's *Divine Love and Wisdom* suggests such a principle: "Many constant things exist, created that inconstant things may exist . . . These . . . [constant] things . . . are provided so that infinitely varying things may exist, for what varies can exist only in what is constant, fixed and certain" (Swedenborg, 1763, para 14). It seems quite possible that the appropriate tension between constancy and change is based on the operation of self-similarity and variety, self-similarity providing constancy and variety inducing change.

CONCLUSION

In this paper we have attempted to show some connection between aspects of religion and of science. This connection took the form of a correspondence relation as illustrated by the properties of self-similarity and variety as found in both religion and science. For our example we focused on the dualistic paradigm of Swedenborg's theological writings and on that subset of science known as chaotic dynamics. Our example suggests common modes of thought and common underlying principles of reality. We conclude by proposing that this exercise demonstrates that the effort to look for a 'seamlessness' in our understanding of ultimate reality and meaning is a challenging but occasionally fruitful endeavor.❑

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