IDEAS FOR SWEDENBORGIAN MATHEMATICAL ILLUSTRATIONS

Neil Simonetti*

ABSTRACT

Emanuel Swedenborg was an eighteenth-century scientist and philosopher before he was called to his use as a theological revelator. His background in science allowed him to draw on this knowledge to illustrate divine truths. Swedenborg states in [11] that "the Divine is everywhere" so this author investigates what analogies Swedenborg may have used if he had a stronger background in mathematics.[1]

1. ASYMPTOTES AND PARABOLAS

As one of the advanced scientists in his day, Swedenborg uses images from science throughout his theological works to illustrate divine truths. He compares the growth of trees and plants to human prolification [20], he uses the heat and light from our sun to illustrate the nature of Divine Love and Divine Wisdom [12], and he draws on his knowledge of the human form frequently to describe the interconnectedness of heavenly systems [16] often referred to as the *Grand Man* [9].

But what if Swedenborg had been a stronger mathematician than a scientist? What kinds of images might he have used to explain divine concepts?

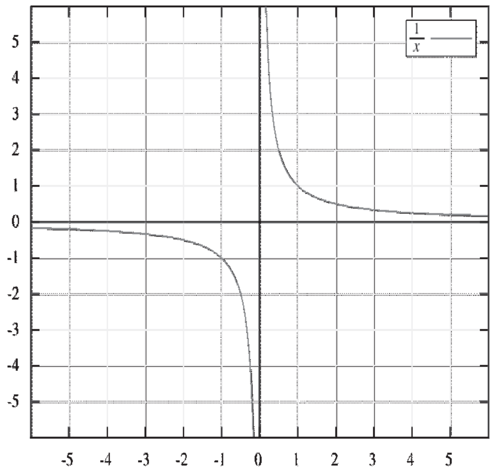
Swedenborg does use concepts in mathematics to illustrate the perfection of angels, but I get the impression that he was not the most comfortable with these concepts. In [18], he states:

---

* Dr. Neil Simonetti is Associate Professor of Mathematics and Computer Science at Bryn Athyn College of the New Church, Bryn Athyn, Pennsylvania. The author is grateful for assistance provided by Robert Cooper.

[1]All pictures are found in Wikimedia Commons: commons.wikimedia.org. Reference to sources are given in square brackets [1].

> I said to the spirits around me that no one, save the Lord alone, is perfect. The angels are not perfect, for heaven is not holy before the Lord [Job xv 15]; nevertheless, the angels can become better and better even to eternity, but they can never become holy in themselves or as to their proprium. Because this seemed strange to the spirits when represented in a spiritual manner, it was therefore elucidated by like things in nature, namely, that there are approximations to infinity, as they are called, which nevertheless do not reach infinity, as for example, between the **asymptotes of the parabola**. But these things must be passed over because they are not understood by many; . . .

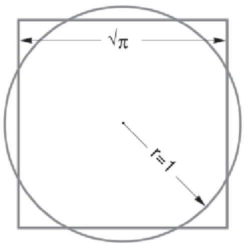


A hyperbola approaching its asymptotes

Is it possible that the mathematical concepts were "passed over" because he didn't have a strong mathematical background himself? Perhaps Swedenborg thought his audience would be more familiar with less abstract analogies in science, so he may have intentionally avoided using abstract mathematical analogies. Swedenborg did make a mathematical error here: parabolas do not have asymptotes, but their close relatives, hyperbolas, do. Swedenborg was very careful not to make such a mistake in a published work, as seen in [15], where he correctly attributes the asymptotic property to the hyperbola when illustrating the continual perfection of angels:

> Although the wisdom of a wise man in heaven increases to eternity, yet there is no such approximation of angelic wisdom to the Divine Wisdom that it can reach it. It may be illustrated by what is said of a **straight line drawn about a hyperbola**, continually approaching but never touching it and by what is said about squaring the circle.

Swedenborg used parts of his *Spiritual Diary* as a rough draft for works intended for publication many times [7], so errors like attributing asymptotes to parabolas left in the *Spiritual Diary* should not alarm Swedenborgian scholars. Swedenborg had summarized the basic mathematical knowledge of his day in his notebook [17], so I can imagine Swedenborg consulting his notebook or one of his mathematician friends before sending his work *Divine Providence* to the publisher.



Squaring the Circle

You may have also noticed his reference to *squaring the circle*, a classical geometrical problem of using a straight edge and compass to produce a square with the same area as a given circle. In Swedenborg's time, there were algorithms that could get close, arbitrarily close with enough effort, but no exact solution was known. It was proven to be impossible in 1882 when Carl Louis Ferdinand von Lindemann discovered that π was not an algebraic number. [5]

2. CONTINUOUS AND DISCRETE

So what opportunities could Swedenborg have taken to bring in mathematical analogies? One opportunity that stands out is his discourse on continuous and discrete degrees, found in [14]:

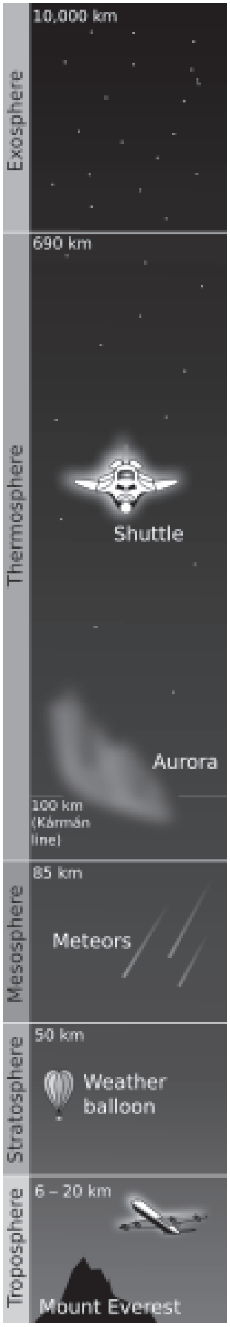
> Continuous degrees are defined as lessenings or decreasings from grosser to finer, or from denser to rarer; or rather as growths and increasings from finer to grosser, or from rarer to denser, exactly like gradations of light to shade, or of heat to cold. Discrete degrees, however, are quite different. They are like things prior, posterior and final, and like end, cause and effect. These degrees are called discrete, because the prior is by itself, the posterior by itself and the final by itself, but yet taken together they make one. The atmospheres from highest to lowest, or from the sun to the earth and which are called ethers and airs, are separated into such degrees. They are like simple things, collections of those, and again collections of these which taken together are called a composite. These

> degrees are discrete because they exist distinctly and these are understood as degrees of height, whereas the former degrees are continuous because they increase continuously, and these are understood as degrees of breadth.

While using gradations of light to shade to illustrate continuous degrees and levels of the atmosphere to illustrate discrete degrees is effective, the number line gives a natural analogy as well. The field of integers, commonly called *whole numbers*, are discrete entities, while the field of real numbers, which include fractions and decimal numbers, are continuous entities.

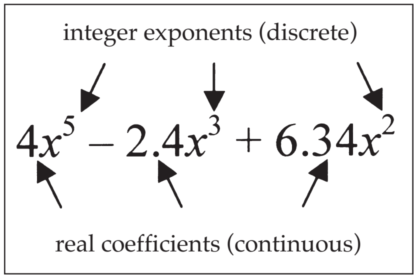
Both systems of numbers are well-ordered, meaning that given any two different numbers, one is "higher" or "larger" than the other. This comparative operator is transitive, meaning that if *a* is larger than *b*, and if *b* is larger than *c*, then *a* must be larger than *c*.

But the two number systems have one key difference which distinguishes the discrete nature of the integers from the continuous nature of the reals. Given an integer, such as 37, you can easily distinguish the next higher integer, 38, just as when you select an atmosphere, such as the stratosphere, there is the next higher atmosphere, the mesosphere. But if you are given a real number, such as 23.7, there is no "next" higher real number. Whatever larger number you choose, such as 23.7000001, there will always be a real number between the two, much like it



appears that there can always be a shade of light between two different degrees of brightness.[2]

To see these concepts working together to make a whole, consider the family of polynomial functions. Each term of a polynomial is discrete, as it can be labeled by its integer exponent, yet each term as a continuous variety by having any real number serve as the coefficient. Both the discrete element and the continuous element are needed to define the family of all polynomials.



An example of a polynomial

Furthermore, if you expand the family of polynomials to include infinitely many terms (but still a countable infinity, since the terms are discrete), then Taylor's Theorem shows that every smooth (infinitely differentiable) function, such as the sine and exponential functions, can be defined as a polynomial, with its continuous and discrete portions.[3] [4]

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \cdots$$

Two functions written as infinite polynomials

3. THE GREATEST AND LEAST

Another concept with an accessible mathematical analogy is found in [13]:

> That the Divine is the same in things greatest and least, may be shown by means of heaven and by means of an angel there. The Divine in the whole

---

[2] It has been noted [2] that while Swedenborg outlined his theological revelations, his knowledge of sciences was not more advanced than the other scholars of his day. The field of physics has changed quite a bit since Swedenborg's time, and the discovery of photons, a discrete particle responsible for electromagnetic radiation including visible light, would allow us to categorize different shades of light and darkness as discrete degrees, not continuous degrees, even though the "distance" between these degrees is smaller than the human eye can distinguish.

[3] In some cases, the intervals on which these infinite polynomials converge to the function may be finite. As an extreme case, it is possible to construct functions where this interval of convergence is a single point.

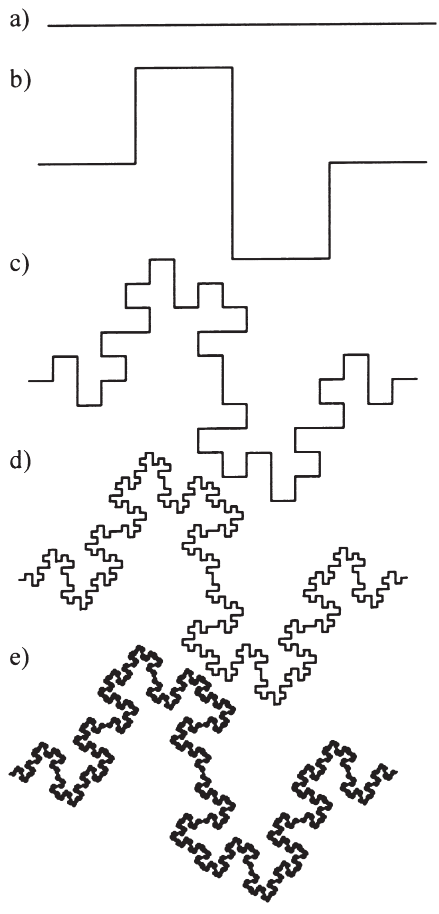
> heaven and the Divine in an angel is the same; therefore even the whole heaven may appear as one angel. So is it with the church, and with a man of the church. The greatest form receptive of the Divine is the whole heaven together with the whole church; the least is an angel of heaven and a man of the church. Sometimes an entire society of heaven has appeared to me as one angel.

The whole of heaven can be seen in the human form. A single society can be seen in the human form. A single person can be seen in the human form. It is the same for the greatest and least things.

Even the functions of a single cell can be organized by systems found in the human body: Lysosomes carry out digestion; the mitochondria act as the muscular system; the cytoskeleton forms the cell's skeletal system and acts as a circulatory system; and centrioles carry out the cell's reproductive activities. [1]

Mathematical objects known as fractals, named for their fractional dimensions, can display this concept extremely well, since the components of a fractal are built by copying the larger structure. No matter how far you "zoom in" on a portion of the fractal, the same patterns continue to emerge.

Consider the Minkowski sausage, named by Mandelbrot to honor this Russian-born mathematician [6], constructed by replacing a single line segment with eight segments each one quarter of the



Making the "Minkowski Sausage"

original's length as shown here. To calculate the dimension of this fractal, determine the ratio of the number of segments changing at each step (one segment is replaced by eight), and the ratio of the size of the segments changing at each step (the old segments are four times as long as the new segments), then divide the logs of these ratios: $^{\log(8)}/_{\log(4)} = 1.5$. Therefore the dimension of the Minkowski sausage is 1.5.

To illustrate the idea that "the Divine is the same in things greatest and least", one can see that looking at any part of this fractal is identical to looking at the entire fractal. There is no loss of detail as you zoom in or out.

4. DIVINE INFINITY

Most people consider infinity to be a quantity without limit. The quantity of positive integers is one example. If you started counting today (1, 2, 3, …) and had an eternity of time to spend, there is no number you would not eventually reach, but you would never be able to count them all. Still, because there is no number that is out of reach, this infinity is called a *countable* infinity by those who study set theory. A *countably infinite set* is a set that can be lined up, one-to-one, with the positive integers. [4]

Examples of countably infinite sets are shown in the following chart. The pattern shows how they may be lined up with the positive integers.

Set	Example										
Positive Integers	1	2	3	4	5	6	7	8	9	10	…
Odd Positive Integers	1	3	5	7	9	11	13	15	17	19	…
All Integers	0	1	–1	2	–2	3	–3	4	–4	5	…
Prime Numbers	2	3	5	7	11	13	17	19	23	27	…
Rational Numbers in (0,1)	$^1/_2$	$^1/_3$	$^2/_3$	$^1/_4$	$^3/_4$	$^1/_5$	$^2/_5$	$^3/_5$	$^4/_5$	$^1/_6$	…
Pairs of Positive Integers	(1,1)	(1,2)	(2,1)	(1,3)	(2,2)	(3,1)	(1,4)	(2,3)	(3,2)	(4,1)	…

Since all of these sets can line up, one-to-one, with each other, mathematicians consider all these sets to be the same size. You might find it odd that that selecting only the odd integers gives a set the same size the all the integers, when common sense would tell you the set should only be half as big. But what would half of infinity look like if it were different from infinity?

Similarly, it is not intuitive that the rational numbers between zero and one would be a countably infinite set, since, unlike the integers, these numbers make up a continuous spectrum. Like the fractals, their pattern is indistinguishable as you "zoom in." Still, by ordering these numbers from smaller to larger denominators, they may be lined up with the set of positive integers.

Swedenborg talks of a *Divine Infinity* as something beyond this notion of infinity in [10]

> People inevitably confuse Divine Infinity with infinity of space. And because they do not conceive of infinity of space as anything other than nothingness, as indeed it is, neither do they believe in Divine Infinity. The same applies to Eternity. They cannot conceive of it except as an eternity of time, but it is manifested continually by means of time to those who dwell within [space and] time.

Is there really an infinity larger than this? That is, is there an infinite set so large that if you attempt to line up the elements of the set, one-to-one, with the positive integers, you are guaranteed to fail, and something will be missed? Swedenborg tries to illustrate this notion in [19]:

> . . . [the Word's] contents are countless, so that not even the angels can exhaust them. Anything found there can be compared to a seed, which planted in the ground can grow into a great tree, and produce an abundance of seeds; these again produce similar trees to form a garden, and their seeds in turn form other gardens, and so on to infinity. The Word of the Lord is like this in its details, and such above all are the Ten Commandments.

This outlines a geometric progression, where if each tree produces, say, ten seeds, each generation would have ten times the seeds as the previous generation. If you number every seed from each tree with a digit from zero to nine, then a seed's genealogy could be represented as a finite decimal number between zero and one. For example, ".142857" would represent one of the sixth generation seeds. These finite strings of digits can also be proven to be countable, since they form a subset of all rational numbers, which can be linked to a subset of all pairs of integers. But the Divine Truth in the Word has infinite depth, so the set of Divine Truths could be compared to decimal numbers which may be infinite in length, such as ".33333333 . . ." or ".14159265. . .". These can be linked, one-to-one, with the set of real numbers between zero and one.

Is the set of real numbers really bigger than the set of rational numbers, which are countable? There are examples of real numbers that are not rational, like π and $\sqrt{2}$. But like comparing odd integers to integers, this does not make the set bigger.

In 1891, Georg Cantor published his diagonalization argument, an elegant proof that shows that the infinity of all real numbers between zero and one is bigger than the infinity of the positive integers. [3] The idea presented is rather simple:

1 – .**5**849204783...
2 – .4**7**59938705...
3 – .50**0**0000000...
4 – .141**5**926535...
5 – .4142**1**35623...
6 – .71828**1**8284...
7 – .618033**9**887...
⋮

n = .6816220...

Cantor's Diagonalization

Suppose that there existed a complete list of real numbers that could be matched, one-to-one with the positive integers. Form a number by taking the digit one higher than the first digit of the first number, then the digit one higher than the second digit of the second number, then the digit one higher than the third digit of the third number, and continue in like fashion. The result is a number between zero and one, call it n. In the example shown, we selected 6, one more than 5, then 8, one more than 7, then 1, and so on. If n were on the list, say in the 724th position, then n's 724th digit would not adhere to the rule for constructing n, requiring that n's 724th digit be the digit one higher than the 724th number's 724th digit. This problem would arise no matter where you attempt to find the number n on our complete list, and so n cannot be in this list, meaning the list is actually not complete. Since we are able to create this n given any attempt at making a complete list, there cannot be a complete list, and therefore the infinity of real numbers from zero to one is bigger than the infinity of positive integers.

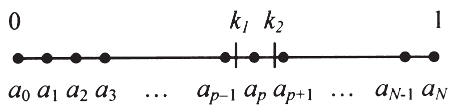
5. OMNIPRESENCE

Related to our infinite friends, the concept of omnipresence can be a little hard to digest. In [8], Swedenborg gives a quick definition: "Omnipresence is infinite circumspection and infinite presence." So how can something be everywhere without constantly getting in the way?

Mathematics has a notion of a *dense set*: Given a numeric interval *J*, a subset of this interval *S* is called *dense* if every subinterval of the original interval *J* contains at least one element from the set *S*. If we choose our interval *J* to be the set of real numbers from zero to one, one might think that the only dense sets possible would have to be the same size. A smaller set, it would seem, could not have the coverage to be "omnipresent", or dense, within the original interval *J*.

As mentioned earlier, the size of *J*, the set of real numbers from 0 to 1, is an infinity larger than the countably infinite set of integers, or the countably infinite set of rational numbers. Yet, the rational numbers are dense within *J*. To prove this, I must show that if I am given an arbitrary interval inside *J*, no matter how small, there must be a rational number contained within this arbitrary interval.

Proposition: The set of rational numbers in the interval from 0 to 1 is dense within the set of real numbers in the interval from 0 to 1.



One element from the sequence, a_p, must be inside the interval.

Proof: Let *K* be an arbitrary interval within the set of real numbers from 0 to 1, and let k_1 and k_2 be the left and right endpoints of the interval *K*, respectively. This makes k_2 bigger than k_1. Let x be the difference, $k_2 - k_1$, of the endpoint values. Think of x as the *width* of the interval *K*. x cannot be zero, otherwise *K* would not be an interval, it would just be a point, so we can assume that $x > 0$. This allows us to choose N to be the next integer larger than $^1/_x$. Since $N > {}^1/_x$, we also know that $x > {}^1/_N$, so $^1/_N$ is a quantity smaller than the width of *K*.

Now define a sequence of numbers a_i by defining $a_i = {}^i/_N$ for each integer *I* from 0 up to *N*. The numbers in this sequence from a set of equally spaced numbers, $^1/_N$ apart, that span the interval from 0 to 1. The first number a_0 is equal to zero, so it is either in the interval K (when $k_1 = 0$) or it is left of the interval K (when $k_1 > 0$). Similarly the last number a_N is equal to one, so it is either in K (when $k_2 = 1$) or it is right of the interval K (when $k_2 < 1$). If none of these points, a_i, were in the interval K, then, since the first one is left of the interval and the last one is right of the interval, the sequence would have to "jump over" the interval K. But since the points in the sequence are spaced more narrowly than the width of the interval K, this is impossible. So the first point of the sequence to pass k_1, call it a_p, must be in the interval K. Since $a_p = {}^p/_N$, a_p is a rational number in the interval K.

This proof can be found in most introductory analysis textbooks, like [4], but more importantly, it shows that the concept of "omnipresence" is not something that Swedenborg made up to describe something that would otherwise be indescribable. ❑

REFERENCES

[1] Campbell, N., J. Reece, L. Mitchell, M. Taylor. *Biology: Concepts and Connections*, 4th edition. (City: Benjamin Cummings, 2003).

[2] Coke, S. "More Canons of Interpretation." *Studia Swedenborgiana* 12 (2001): 2. 12:2.

[3] Dauben, J. *Georg Cantor: His Mathematics and Philosophy of the Infinite*. (Princeton: Princeton University Press, 1979).

[4] Gaughan, E. *Introduction to Analysis*, 4th edition. (Pacific Grove, CA: Brooks/Cole Publishing Company, 1993).

[5] James, G. *Mathematics Dictionary*, 4th edition. (New York: Van Nostrand Reinhold Co, 1976).

[6] Lauwerier, H. (transl. S. Gill-Hoffstadt) *Fractals*, Princeton: Princeton University Press, 1991

[7] Odhner, G. The Crossed-Out Passages in the Spiritual Diary, *New Church Life, 97* (1977): 612–614.

[8] Swedenborg, E. (transl. Whitehead) *Apocalypse Explained*, § 282.

[9] _______________ (transl. Elliott) *Arcana Coelestia*, § 550.

[10] ______________ (transl. Elliott) *Arcana Coelestia*, § 1382.

[11] ______________ (transl. J. Ager) *Divine Love and Wisdom*, § 8.

[12] ______________ (transl. J. Ager) *Divine Love and Wisdom*, § 12.

[13] ______________ (transl. J. Ager) *Divine Love and Wisdom*, § 79.

[14] ______________ (transl. J. Ager) *Divine Love and Wisdom*, § 184.

[15] _______________(transl. Dick and Pulsford) *Divine Providence*, § 335.3.

[16] ______________ (transl. Harley) *Heaven and Hell*, § 59.
[17] ______________ (transl. Acton) *A Philosopher's Notebook*.
[18] ______________ (transl. Buss) *Spiritual Diary*, § 883.
[19] ______________ (transl. Chadwick) *True Christian Religion*, § 290.
[20] ______________ (transl. Chadwick) *True Christian Religion*, § 585.