CHANCE IN MODERN SCIENCE

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Before beginning my paper proper I want to make as clear as possible what I mean by chance-in-principle. I mean a philosophical conclusion arrived at critically by only a few and adopted without much analysis by many. It is my belief that it affects the education of almost all who study science today, either on an elementary or an advanced level, whether they realize it or not.

> Chance-in-principle is a belief that the success in the application of statistics or the theory of probability results because nature acts according to laws of chance—a belief that the laws of chance and the laws of nature are one and the same.

There is another philosophical view concerning statistics—the one held by those who first applied statistics in physics during the 19th Century. This was the belief that statistics apply where the number of systems involved is large and where the detailed knowledge concerning the systems is insufficient or the complexity is such as to prevent the application of individual cause-and-effect relations. I call this last interpretation chance-in-appearance to distinguish it from chance-in-principle.

My remarks will consist of four parts. The **Introduction** will briefly recite how "chance" came into the scientific picture and *that* but not *how* statistics have developed over the years. Because of technical difficulties I cannot tell you how physics came to depend upon statistics to such an extent in quantum mechanics, but in **Part II** I will try to give you in their own words some of the ideas of the creators of modern physics on the pros and cons of chance-in-principle.

There is a corollary to the chance-in-principle concept, that teleological or purposeful explanations of nature ought to be discarded. I shall give an example using biology and the history of architecture illustrating this in **Part III.** And finally after this somewhat negative approach in Parts II and III, as a conclusion in **Part IV,** I shall try to recite some gains or good effects that may be derived from the elements referred to in Parts II and III.

Introduction

I was asked about the word "chance" in the title of this paper. Why not "accident"? It is true that the laws of chance are applied to what are termed accidents. But this in no way exhausts the objects of study to which the laws of chance are applied.

The original application of the mathematical theory of probability over 200 years ago was to games of chance. In fact it was for this limited purpose that the elements of the mathematical theory of probability were developed.

The principal figures involved in this episode during the 17th century were Pascal and Fermat, two mathematicians, and a French nobleman, Chevalier de Mere, "A man of ability and great experience in gambling." (See *Introduction to Mathematical Probability* —Uspensky. The introduction includes a section "Short Sketch of Historical Development of Theory of Probability.") After development by the Bernoullis came De Moivre, who in 1718 published his work, *The Doctrine of Chance.* It might be because of the popularity of this book and its subsequent republication in 1738 and 1758 that the term "chance" has continued to be associated with the applications of probability and statistics.

At any rate it is common practice to speak of the applications of mathematical statistics and the theory of probability to problems in science as the laws of chance. This is analogous to the practice of referring to the application by Newton of the calculus to mechanical problems in nature as the laws of mechanics.

Perhaps the best known early application of the laws of chance in science was by the Augustinian monk Gregor Mendel, whose report on his experimental work on dominant and recessive types in the common pea plant was published in 1865.

By the turn of this century the classical statistics associated with the names of Boltzmann and Gibbs were fully developed. These statistics are the bases upon which thermodynamics and the kinetic theory of gases were developed.

Already by this time the opinion that statistics in principle, or better, the concept that the law of chance-in-principle is the way in which creation operates, was taking root in the life sciences, as the Darwinian theory of evolution developed into what is termed emergent evolution. However, at this time, that is at the turn of the century, the application of classical statistics in physics was seldom regarded as a law of nature in principle.

For example, one applied statistical reasoning in the kinetic theory of matter in lieu of exact knowledge, as a substitute for precise cause-and-effect information where it was lacking. The best known examples of an application of this sort are found in first-year college science text books. Statistical considerations are applied to the numerous molecules in a volume of a so-called perfect gas, and as a result, certain empirical laws are obtained—Boyle's Law, for example.

One must acknowledge the successes of the application of probability to a multitude of problems in science. But there are different philosophical conclusions with regard to the meaning of such successes. The question is: are we to accept in principle the laws of chance as the laws of nature, the means by which creation operates? Or are we to accept the idea that these laws of chance are a substitute for exact knowledge, the lack of which implies causes hidden in the deeper regions of nature? These are the two alternate choices open to us.

At present the interpretation by the emergent evolutionist that the laws of chance are laws in principle has passed over to the physical scientist. The circumstances which separated the interpretations of probability by the emergent evolutionist and the physicist in the early years of this century no longer exist.

During the intervening years considerable additions have been made to the theory of statistics. Even the fundamental properties of the elementary particles of nature such as the electron, the proton, the neutron, and so forth, are identified in terms of statistics. In 1924 the Indian, Sir Jagadis Chandra Bose, Albert Einstein, and Louis de Broglie developed a special form of statistics to be applied to particles which are indistinguishable, for example alpha particles. And a little later Enrico Fermi and P. A. M. Dirac developed a complementary statistics to be applied to distinguishable particles —for example electrons with opposite spin. The former of these statistics came to be known as Bose-Einstein statistics, and they are applied to photons and neutral helium atoms as well as alpha particles. Such particles are called "bosons." The other statistics, called Fermi-Dirac statistics, apply to electrons, protons, neutrons, and pi mesons, and these are called "fermions."

This illustrates only a part of the development in statistics and their interpretation. Chief among the new ideas is the substitution in quantum wave mechanics of probability functions for those functions which in Newtonian mechanics gave precise distance as a function of time, such as, for example, the distance s a freely falling body moves in time t, known to high school students of physics,

$$s = \frac{1}{2} gt^2.$$

In wave mechanics no such relation exists. Instead, when one solves for a distance the result is expressed as a "psi-function," as it is called. The psi-function itself is not used. However, its square, or the product of it multiplied by itself, is interpreted as the probability that a particle will be within a given region. Many philosophical discussions that have taken place in physics during the last thirty years have centered upon this psi-function.

Part II

Having introduced this brief history of the development of statistics, I now turn to the second part, wherein I shall try to give some idea how the leading physicists who have been concerned with the philosophical problem of chance-in-principle have expressed themselves.

The question is not, shall we use statistics or shall we not? The question is, how do we interpret the meaning of statistics when we do use them? That is, do the results of statistics represent the results of the laws of nature themselves, or are these the results of statistically significant sampling in the absence of exact knowledge? The former of these represents a philosophical judgment about the cosmology, the structure of creation. The second view is that of a stratagem to get around ignorance or through complexity and to settle for the best answer available under the circumstances.

I wish to give some of the atmosphere of the discussions and the disagreements that have taken place between the leading physicists concerning the interpretation of statistical results. One would expect efficient and successful agreement among the best minds.

That they do not agree, even after numerous exchanges in books, congresses, and letters, indicates strongly that there must be something deeper to constructs of human thought than can be shown by clearly definable methods of science.

Max Born defines the problem as follows:

> Dynamical problems in quantum theory therefore, in contrast to those in classical theory, cannot be defined without a subjective, more or less arbitrary decision about what you are interested in. In other words, quantum mechanics does not describe an objective state in an independent external world, but the aspect of this world gained by considering it from a certain subjective standpoint, or with certain experimental means and arrangements. This statement has produced much controversy, and though it is generally accepted by the present generation of physicists it has been decidedly rejected by just those two men who have done more for the creation of quantum physics than anyone else, Planck and Einstein. Yet, with all respect, I cannot agree with them. In fact, the assumption of absolute observability which is the root of the classical concepts seems to me only to exist in imagination, as a postulate which cannot be satisfied in reality. (*Natural Philosophy of Cause and Chance*, Born, pp. 99–100.)

The purpose of this selection here is to point out the division of Einstein and Planck from the others. There are many such references in the writings of several leading physicists. But I must restrict my references in this respect and I will stay with Max Born, because evidently he has been the most effective influence in arguing in favor of the chance-in-principle doctrine.

In at least two places Born considered the contents of a letter he received from Einstein to be important enough to quote verbatim in English translation. Born says:

> I translate here a passage of a letter from [Einstein] which I received about four years ago (7th November, 1949): "In our scientific expectation we have progressed towards antipodes. You believe in the dice-playing god, and I in the perfect rule of law in a world of something objectively existing which I try to catch in a wildly speculative way. I hope that somebody will find a more realistic way, or a more tangible foundation for such a conception than that which is given to me. The great initial success of quantum theory cannot convert me to believe in that fundamental game of dice."

And from an earlier letter of December 3, 1947, Max Born translates:

"I cannot substantiate my attitude to physics in such a manner that you would find it in any way rational. I see of course that the statistical interpretation (the necessity of which in the frame of the existing formalism has been first clearly recognized by yourself) has a considerable content of truth. Yet I cannot seriously believe it because the theory is inconsistent with the principle that physics has to represent a reality in space and time without phantom actions over distances. . . . I am absolutely convinced that one will eventually arrive at a theory in which the objects connected by laws are not probabilities, but conceived facts, as one took for granted only a short time ago. However, I cannot provide logical arguments for my conviction, but can only call on my little finger as a witness, which cannot claim any authority to be respected outside my own skin." (*Ibid.*, p. 123.)

What are Born's reactions to these remarks of Einstein? Among other things he says:

[Einstein] obviously agrees that we have at present nothing better, but he hopes that this will be achieved later, for he rejects the "dice-playing god." I have discussed the chances of a return to determinism and found them slight. I have tried to show that classical physics is involved in no less formidable conceptional difficulties and had eventually to incorporate chance in its system. We mortals have to play dice anyhow if we wish to deal with atomic systems. Einstein's principle of the existence of an objective real world is therefore rather academic. (*Ibid.*, p. 123.)

But a much deeper observation by Born is the following:

Einstein's letters teach us impressively the fact that even an exact science like physics is based on fundamental beliefs. The words *ich glaube* appear repeatedly, and once they are underlined. I shall not further discuss the difference between Einstein's principles and those I have tried to extract from the history of physics up to the present day. But I wish to collect some of the fundamental assumptions which cannot be further reduced but have to be accepted by an act of faith. (*Ibid.*, p. 123.)

There are a few in the younger generation who have not given up a faith in traditional concepts. One of these is David Bohm, who is very concerned with the problem of re-establishing cause as a more fundamental concept than that of chance. His confidence that this can be done rests in a belief that there are levels in creation beyond those which have made themselves known in experiments.

In a foreword to Bohm's book *Causality and Chance in Modern Physics* Louis de Broglie, one of the founders of wave mechanics who has been more like Einstein in his attitude toward cause than like Born, says the following:

> The construction of purely probabilistic formulae that all theoreticians use today was thus completely justified. However, the majority of them, often under the influence of preconceived ideas derived from positivist doctrine, have thought that they could go further and assert that the uncertain and incomplete character of the knowledge that experiment at its present stage gives us about what really happens in microphysics is the result of a real indeterminacy of the physical states and of their evolution. Such an extrapolation does not appear in any way to be justified. It is possible that looking into the future to a deeper level of physical reality we will be able to interpret the laws of probability and quantum physics as being the statistical results of the development of completely determined values of variables which are at present hidden from us. It may be that the powerful means we are beginning to use to break up the structure of the nucleus and to make new particles appear will give us one day a direct knowledge which we do not now have of this deeper level. To try to stop all attempts to pass beyond the present viewpoint of quantum physics could be very dangerous for the progress of science and would furthermore be contrary to the lessons we may learn from the history of science. This teaches us, in effect, that the actual state of our knowledge is always provisional and that there must be, beyond what is actually known, immense new regions to discover. Besides, quantum physics has found itself for several years tackling problems which it has not been able to solve and seems to have arrived at a dead end. This situation suggests strongly that an effort to modify the framework of ideas in which quantum physics has voluntarily wrapped itself would be valuable. (*Causality and Chance in Modern Physics*, Bohm, pp. IX–X.)

There are many who claim that this effort of Bohm and others to see cause on deeper levels of nature is doomed to failure. The theorem of von Neumann is often quoted against Bohm. It proves that there are no hidden variables and that wave mechanics is a complete system. And yet the leaders continue to have their conferences on the subject and so they do not seem to feel the question is answered. Max Born himself is a living testimonial that the statistical interpretation is not as certain as one might gather from reading some of his statements. The fact is that, long after the question is supposed to have been settled back in 1928–30, he continues to write about it—with slight modifications here and there. Although this testimonial is perhaps one peculiar to myself, I think considerable substance could be given to it by reciting a large number of remarks made by Born in various places.

Nevertheless, beyond the logical conclusions of von Neumann and others based upon the fundamental assumptions of quantum mechanics, there are other thoughtful attitudes of von Neumann and these others. In chapter III of his *Mathematical Foundations of Quantum Mechanics* he identifies classical mechanics—that is, Newtonian mechanics—with causality. He considers the possibility raised by David Bohm and others that the dilemmas in physics that have brought about statistical interpretations instead of those of mechanistic determination could be resolved by looking to deeper levels in nature for what are termed "hidden variables": He says,

> Whether or not an explanation of this type, by means of hidden parameters, is possible for quantum mechanics, is a much discussed question. The view that it will sometime be answered in the affirmative has at present prominent representatives. If it were correct, it would brand the present form of the theory as provisional, since then the description of the states would be essentially incomplete. (pp. 209–10.)

And he concludes:

> This concept of quantum mechanics, which accepts its statistical expression as the actual form of the laws of nature, and which abandons the principle of causality, is the so-called statistical interpretation. It is due to M. Born, and is the only consistently enforceable interpretation of quantum mechanics today—*i.e.*, of the sum of our experiences relative to the elementary processes. (p. 210.)

After a considerable formulation of a statistical nature, more than a hundred pages later von Neumann returns to causality considerations. He makes this significant observation:

> The question of causality could be put to a true test only in the atom, in the elementary processes themselves, and here everything in the present state of our knowledge militates against it. (p. 327.)

Here is a clear example of a demand that the application of discrete degrees to this part of creation and its relation to the macroscopic be examined. If this is a true representation of discrete degrees, then herein lies an illustration of the dependence of cause upon discrete degrees as a prior concept.

Part III

I now proceed to the third part of my discussion, where I will give an example of another philosophical judgment that goes hand-in-hand with the chance-in-principle concept of statistics. This is the challenge against the concept of purpose in creation. The philosophical concept of purpose goes by the term teleology.

The chance principle has been accepted by both physicists and biologists. But the main arguments concerning purpose are to be found more as one approaches the life sciences than in the physical sciences.

A sample is given by Harold Blum in his *Time's Arrow and Evolution.* The theme of this book as indicated in the preface is based upon the acceptance of the second law of thermodynamics as a universal law of nature. It is because this law prescribes that there be certain irreversible processes in nature that the book has its title *Time's Arrow and Evolution.* Although the book seems to have been about twenty years in preparation, from about 1933 to 1950, a time when enormous changes were taking place in science, the author states:

> There has been nothing in all this change, however, to weaken the status of the second law of thermodynamics; rather its general applicability to the evolution of the nonliving world—and to that of the living world as well—has become more obvious. (Preface vii, 1950.)

By the time a new edition was printed the author states:

> The need to take the second law of thermodynamics into our thinking about evolution appears to me all the more certain; although I am less sure that I have said so as clearly as I could wish. (Preface ix, 1954.)

With such a theme the author proceeds to give what appears to him to be an explanation of the evolutionary process leading from earliest times up to the present. At the end of his book he has a section devoted to the subject of teleology. The entire argument is contrary to any belief in the possibility of purpose of any sort.

Environment being what it is, a living organism once formed undergoes slight changes or mutations. Those organisms whose mutations result in a better fit to environment persist; others perish. As he says, This is an active process in that the system concerned changes to fit its environment—not *in order to* fit its environment, but as a result of mutation and natural selection. (p. 207.)

And a little later on he says,

Order we find, and fitness, and evidence that order and fitness have been arrived at by an evolutionary process, which we make some shift to explain. (*Ibid.*, p. 210.)

Previously the author has told a story, the story of the development of architecture during the Romanesque and up through the Gothic period. Here he says,

Looking back today, with the perspective of several intervening centuries, we may trace an orderly evolution from the one form to the other, in which short-lived divergences from the main channel do not confuse us. May we find some analogy here to the evolution of living organisms? In both instances each new step was dependent upon those that had preceded, but there seems very little evidence, even in the architectural analogy, that the final achievement was foreseen in advance. (p. 209.)

That is, the planner of the Romanesque did not foresee the Gothic —nor even did the builder of foundations nor its planner often live to see the stone Gothic arches set in place. He says,

But did the architects who began the movement away from the old Romanesque in the directions of what would many years later be the new Gothic really have their goal in sight?

And so we return to his remarks about organic evolution:

Similarly, we may see in organics semblances of purpose and of progress toward a goal, but the architectural analogy must give us cause for skepticism. (p. 210.)

Such is a philosophical judgment of the naturalist. Yet even he cannot go all the way in depending completely upon pure chance to explain the mechanism of evolution. Evidently there is a missing intellectual link to this argument. For example we note near the middle of the book, in a section entitled "Stability and Variability," the following modification of the pure chance explanation. He says,

> These variations or mutations would not occur in a purely random fashion but would be restricted according to the configuration of the molecule or molecules constituting the gene. . . . Whatever the nature of mutation, it will have to follow certain lines that are determined by molecular pattern and energetic relationships. Mutations, then, are not random, but may occur only within certain restricting limits and according to certain pathways determined by thermodynamic properties of the system. Thus, to state the case in a somewhat animistic fashion, the organism cannot fit itself to the environment by varying unrestrictedly in any direction. Certainly Darwin and many who have followed him have thought of variation as completely random, and hence providing all possible variety of handles for natural selection. In many ways this is still a good working point of view, but it may lead to difficulties if carried to extremes. The outcome of such ability to vary in all directions would be that, given time and opportunity, the organism might eventually vary in such a way as to fit itself to its environment with a high, or indeed a perfect, degree of accuracy. (p. 155.)

It seems quite unnecessary under the circumstances to take the stand that nature does not give a picture of purpose. And yet it remains a provocative problem to the New Church philosopher to examine these arguments in detail. As one follows through the arguments, not only in quantum mechanics but also in evolution, one meets numerous Y-branches in the road. By carefully choosing the right branch in each case, one can indeed arrive at a terminus that denies cause or denies purpose itself.

But I ask you, does not this very process itself exemplify purpose? The reason for choosing this path would appear to be the objective set up in the first case, namely, to prove that teleology does not exist, or at least that explanations are possible without teleology. Having made the assumption, is it not this very assumption which dictates the choice at each place where the road divides?

Certainly it seems there are other evolutions than architectural or biological, in which we might examine the analogy used by Blum. Consider, for example, the evolution of a science, say physics, during the last sixty years. Certainly those who worked in this field for the first three decades had no way of foreseeing what was to happen in the next three. The atomic physicists could scarcely have any idea that their developments would lead to nuclear physics —still less could they have any idea of what this subject would be in detail. Nevertheless the foundation for this was laid at the very beginning of these sixty years by Planck with his hypothesis concerning the quantum, by Einstein with his hypothesis about the application of the energy-mass relation among other things, and by Rutherford in alpha-particle experiments.

No creative scientist, no creative mind in any field, can know what tomorrow will bring. No one of them has to wait for as long as 60 years for astonishing, unforeseen things to happen. Is this to say that in the evolution of physics of the last 60 years, in the creation of the scientist, from day to day, there is nothing that is with purpose? I am sure that given enough wits and time, and by setting out with the purpose to show that teleological explanations are not necessary in any of these, one can do so.

Recently I was struck with the idea that if some physicists and biologists are choosing the pathways of their arguments to suit their ends, others may be doing so too. I happened to run across this remark about the study of history in a book by a chemist, A. R. Patton, entitled *Science for the Non-Scientist.*

> Nowadays when many seem to hold the view that a scientist who dares to use the word *God* is out of bounds, it may seem surprising to learn that presuppositions which led to the new science, beginning in the seventeenth century and continuing to this day, were rooted in a firm belief in one God. In fact, there is ample historic documentation for the statement that modern science was an outgrowth of the Christianity of the time; although we shall not attempt to present such a case here. Although it is now possible to point out that the phenomena of the physical world might conceivably have grown out of Atheism, an examination of the lives and beliefs of the great founders of scientific thought will show that such an idea simply did not occur to them. (p. 8.)

Evidently if what Patton says is correct, then as in biology or physics so in history one can in freedom make choices, which determine the end result or the conclusion.

Part IV—Conclusion

Much of what has been said in these remarks may be regarded as negative, but this applies only to my attitude with regard to the chance-in-principle interpretation of the successes of statistics. There is much in the history of physics of the last sixty years that merits positive consideration in its possible relation to New Church philosophy. I shall recite six such points.

First: There is the strong evidence of the existence of discrete degrees. Seeds seem to have been sown that will grow into a belief in the existence of a reality beyond the senses.

Whether scientists as a group will ever come to this conclusion no one can say. But already there are some who follow Einstein and others in the firm belief in cause and effect relations. Because the arguments are all against a return to the nineteenth century mechanistic determinism, these people are looking for causal explanations on a deeper plane. This would be consistent with the doctrine that relates cause and effect to discrete degrees.

Second: It must be recognized that New Church arguments cannot be brought to bear in opposition to chance as an appearance. Rather we should read the main arguments of the Writings as warning us away from viewing things in creation as happening according to chance. Nevertheless it is of permission that we do view much as if happening by chance in order that man's freedom may be preserved. According to the doctrine of permissions, some things are allowed in order to prevent a more grievous error, in this case profanation.

The Writings say, for example:

> The Divine Providence works thus invisibly and incomprehensibly in order that man may in freedom ascribe an event either to providence or to chance; for if providence acted visibly and comprehensibly, there would be danger of man's believing, from what he sees and comprehends, that it is of providence, and afterward changing into the contrary. (AC 5508.)

The present view in a world of chance-in-principle may therefore be of Divine permission in order to prevent more grievous errors. But I am not prepared to go further with this idea.

Third: The history of the last sixty years contains what seems to me to be a certain virtue. By contrasting the probabilistic successes with the deterministic failures, the arguments in favor of mechanistic materialism have been seriously weakened. This 19th-century philosophy had all but swept through the sciences by the beginning of this century. And one must realize that many of the arguments against cause and effect among scientists today are raised more against this philosophy than against such a one as that held by the New Church. One must remember that at the inception of this materialism one of its leading protagonists, Laplace, is supposed to have answered Napoleon when asked by him why God is not mentioned in Laplace's work *System of the Worlds*—"Sire, I have no need for that hypothesis."

Fourth: Those of us over fifty have seen the error of identifying religious ideas with scientific ones. If for example it was an error to identify the concepts of fatalism with mechanistic determination and hence also predestination, then this historical experience warned us that free will could not be made to depend upon the indeterminacy principle and chance-like possibilities. Indeed the effort to do so by prominent thinkers, especially in England, during the late twenties and early thirties was short-lived.

Fifth: There is the increased emphasis upon the subjective nature of science. Certainly if there is one thing that is evident in the history of physics in the last sixty years it is the increased realization of the subjectivity that is in science. I refer not only to Born's comments about Einstein's *Ich glaube*—I believe—but I refer also to the subjectivity that is inherent in physical measurements themselves. And I do not refer here to the well-known measurement errors. I refer to something that lies at the very basis of quantum mechanics that has to do with the effect of measurement itself upon the object measured. And in return there is the reaction of the instrument to what is being measured. This is subjectivity of a fundamental nature which in the study of the elementary particles of nature cannot be avoided. Its critical evaluation is accomplished under what is known as the indeterminacy principle, which of course I cannot go into here.

In the macroscopic world the subjective action and reaction mentioned above can be made as small as is practically necessary and so it can be ignored. In this sense, and in only this sense can Newtonian mechanics and all of science which depends upon that mechanics be made objective. And so in the macroscopic world we have such "exact" laws as $s = \frac{1}{2} gt^2$.

But in the microscopic world this subjectivity cannot be ignored. This is apparently a principle. If the location of an electron, for example, and its momentum cannot both be exactly measured at the same time, then no amount of mathematics can determine where it will be at some future time. If cause has been identified with such a form of determinism, then such a cause is not of fundamental nature. As Heisenberg has been quoted:

But in the formulation of the causal law, namely, "If we know the present exactly, we can predict the future," not the conclusion but the premise is false. (*Atoms, Molecules and Quanta,* Ruark & Urey, p. 622.)

Sixth and finally: There is one important lesson to be learned from the experience of the physicists, and that is that it appears that traditional philosophical concepts cannot be avoided. If specialists in other fields have been able to carve for themselves slices of interest narrow enough to include only questions they can answer and thus satisfy the pragmatic philosophy of the age, fortunately for science and philosophy the quantum theorist of the first rank has not been able to do so.

I cannot help before closing to call your attention, because your presence implies interest in philosophy, to a frank statement by Werner Heisenberg concerning the importance of the problems of traditional philosophy to the study of science.

To mold our thoughts and language to agree with the observed facts of atomic physics is a very difficult task, as it was in the case of the relativity theory. In the case of the latter, it proved advantageous to return to the older philosophical discussions of the problems of space and time. In the same way it is now profitable to review the fundamental discussions, so important for epistemology, of the difficulty of separating the subjective and objective aspects of the world. Many of the abstractions that are characteristic of modern theoretical physics are to be found discussed in the philosophy of past centuries. At that time these abstractions could be disregarded as mere mental exercises by those scientists whose only concern was with reality, but today we are compelled by the refinements of experimental art to consider them seriously. (*The Principles of the Quantum Theory,* Werner Heisenberg.)