SWEDENBORG SCIENTIFIC ASSOCIATION

The Sixty-Ninth Annual Meeting of the Swedenborg Scientific Association will be held in Bryn Athyn, Pennsylvania, at the Civic and Social Club at 8:00 p.m., Sunday, May 8, 1966. The meeting will be preceded by a supper at 7:00 p.m. ($1.00).

There will be brief reports and election of President and members of the Board of Directors. Nominations for these elections are listed on the back cover of this issue of the NEW PHILOSOPHY.

The Reverend Ormond Odhner will deliver the annual address, entitled: "Two Sources of Truth—or Two Foundations."

All are welcome.

PHILOSOPHICAL NOTES

History and Symbols. Speaking at the meeting of the Swedenborg Scientific Association in May, 1965, Dr. Hugo Lj. Odhner said:

> The pragmatic modern mind—overloaded with facts—is easily made contemptuous of metaphor, and underestimates the value of what the Writings call *representative truth,* the type of truth which speaks in correspondences and allegories, and is addressed rather to the heart than to the brain. Yet this type of truth is the first form of human communication—older than speech or words. The men of the "Golden Age" are said to have communicated by gestures, actions, and sounds rather than by articulate words. And the power of rituals, representing spiritual powers and spiritual processes of salvation from evil, became in time abused and turned into magical practices; while underlying truths were turned into myths no longer understood. (NEW PHILOSOPHY, July–Sept., 1965, p. 83.)

I have already referred to this remark in previous notes, but I repeat it so that readers of this issue will have it handy. There is an accumulation of words in this remark that have challenged me to do some thinking, not only about metaphor in particular but more generally about symbols—and specifically what I have elected to call radical symbols.

Introductory Note on Symbols. In the July–Sept. issue of the NEW PHILOSOPHY for 1965 a discussion was begun in these notes on symbols, with a reference to Metaphor and the Modern Mind. The discussion has grown longer than was then intended.

It was then intended only to indicate other possible symbols than the metaphor. It was also intended then to close off the discussion with a brief comment on positivism. However, it soon became apparent that these notes could be the vehicle for introduction to a vastly more general and considerably deeper problem than was immediately involved in the above two modest intentions.

Not only are there other verbal or linguistic symbols than the metaphor, such as the aphorism, dialogue, epic, etc. There are other symbols in kind, as in art, mathematics, etc.

The relation of symbols to the question of philosophical monism or dualism grows as we pursue the notes. In previous notes this relation was restricted to the question of whether a representation is a mere extension of its object or something quite distinct. However, at the very outset there was the recognition obtained from Wheelwright that what he called the radical metaphor could not be reduced to a simile.

By generalizing this use of the term "radical" as I have done, by applying it to symbols other than metaphors, I am suggesting these symbols may not be reducible to each other. The suggestion goes further, viz., that each symbol represents distinctive knowledge about creation, that the knowledge that comes to us through the different symbols represents different aspects of creation.

This brings us to the very brink of opposing a point of view that has been supported since William of Occam (c. 1280–c. 1347). This is the point of view known as nominalism. His words have become well known: *Entia non multiplicanda praeter necessitatem.* That is, entities or principles should not be unnecessarily multiplied. The principle is so well known and applied so often in philosophy and other disciplines, especially natural science, that the whole philosophy of nominalism is recognized by the term "Occam's Razor." (See for example *Ideas Have Consequences* by Richard M. Weaver, for an interesting account of the influence of nominalism. For historical development and special application of "Occam's Razor" to nominalism see *Medieval Philosophy* by Maurer, pp. 284–5.)

If we apply this doctrine to the disciplines of psychology, biology, physics, mathematics, and logic, we have what is known as reduction. Mathematics can be reduced to logic, physics to mathematics, etc.

Present-day philosophy of education and of science is filled with nominalism. Thomas H. Huxley (1825–1895) wrote an essay "On the Educational Value of the Natural History Sciences." In it he states it is his purpose to "consider in succession:

1. [Biology's] position and scope as a branch of knowledge
2. Its value as a means of mental discipline
3. Its worth as practical information

And lastly,

4. At what period it may best be made a branch of Education."

(Reprinted in *Readings in Philosophy of Science,* Ed. by Philip P. Wiener, pp. 127 f.)

Clearly we have here an example of an effort to integrate in one essay a doctrine that has epistemological, ontological, value, and educational consequences.

Huxley discusses the relation as to method between the life science, biology, on the one hand, and mathematics, chemistry and other natural sciences dealing with unalterable or dead objects that preserve their *"quantity* and *figure,"* on the other. He comes to the conclusion that all apparent differences in method between the disciplines vanish under his analysis. He says: "No such differences, I believe, really exist." There are differences in subject matter but not in method. Here is a sweeping effort at reduction.

Another very prominent illustration is the Russell-Whitehead effort to reduce mathematics to logic. By reduction I do not here refer to correlation of knowledge nor its accumulation by encyclopedic or other schematic arrangements. I refer to something deeper.

The effort to unify method passes over into an effort to unify reality itself. This is monism. There are various representations of monism, notably idealism and materialism. There are still other forms held to by people who are essentially nominalistic in their view but who cannot accept idealism or materialism. One of the best known examples is Bertrand Russell. He sums his ideas in one place in these words:

To show that the traditional separation between physics and psychology, mind and matter, is not metaphysically defensible, will be one of the purposes of this work; but the two will be brought together, not by subordinating either to the other, but by displaying each as a logical structure composed of what, following Dr. H. M. Sheffer, we shall call "neutral stuff." (*The Analysis of Matter,* Chapter I, end.)

What Russell really believes as to the nature of things is sometimes difficult to discover. In fairness to him both in this place and others where his name is used in the notes, it should be stated that although he is known as philosopher, mathematician, and other things, the predominating influence in his writings is logic. At any rate he continues with respect to the above as follows:

We shall not contend that there are demonstrative grounds in favour of this construction, but only that it is recommended by the usual scientific grounds of economy and comprehensiveness of theoretical explanation.

Whatever this means in other respects, we see Occam's Razor appearing in the words "usual scientific economy," and reduction in the words "comprehensiveness of theoretical explanation."

Program for This Set of Notes. I will add some general notes on symbols, re-emphasizing what is meant by radical symbol, the possible relation to monism and pluralism. Also there is a note introducing the possible relation of "reduction" to the "radical symbol." After some notes relating mathematics to logic I will add something on mathematics to illustrate its radical property: that is, how we can produce new knowledge through its use. In this set I restrict myself to algebraic equations. In a future set of notes I think it will be useful to extend this program through the example of differential equations. Finally this set of the notes will be closed with some references that are distinctive to the Writings of Emanuel Swedenborg on significative and representative symbols, and on correspondence.

The reader who has followed me since last July will probably wonder whether something will be done with myth, with positivism, and with the philosophy of history—all topics related to the remarks in Hugo Lj. Odhner's paper that stimulated all this note writing. I hope we shall see something on these.

Symbols. One or two readers have asked me if I have read such and such a book on symbolism. It was never my intent to discuss symbolism in the usual colloquial meaning of the term. The metaphor has deep significance beyond the special symbols that have been created through the grammatical metaphor. The aphorism at one time had a deeper meaning than any particular gnomic statement created by a wit. And so it goes for poetry, for the epic, for the myth, and for many other symbolical forms. Many symbols have been created in art—in drawing and painting, in sculpture, and in music, and each of these seems to have an irreducible or radical character.

As the notes have evolved it has been my intention to develop this deeper approach to the possible meaning of symbol, deeper than one can arrive at by considering a special kind of symbol. As an example of one kind there are many verbal forms such as aphorism, the dialogue, the epic, etc. Each of these is a complex or a compound symbol using word forms.

As an example of another kind there are mathematical forms, whether they be the hieroglyphic representations of the *Rhind Papyrus;* the arguments and reasons of Euclid; the literal equations of Diophantus demanding integral solutions; the modern algebraic equations admitting of "algebraic" numbers, positive and negative, rational and irrational, real or complex; the differential equations stemming from Newton and Maxwell; the kind of differential equations originated by Schroedinger; etc.

Still another example is found in art forms, whether they be simple objects such as candelabra or images; human or animal figures; simple representatives or paintings; architectural representations such as the Ark or The Tabernacle of the Old Testament; or a modern complex including painting, sculpture, and architecture, examples of what have come to be called "the fine arts."

Just as there are these visual forms, word forms, and mathematical forms, there are also acoustical forms such as music and combinational forms such as the dance. Blind people no doubt know about sensual forms represented in touch.

A very simple illustration of the symbol in its various forms or representations is the word "victory." Those who had reached maturity in 1940 will recall the symbol of the opening phrase of Beethoven's Fifth Symphony. Sometimes in the news events of that day one could not disentangle this musical phrase from the symbol "Churchill." Sometimes both the music and the man were replaced by a simple symbol, the letter V. But best of all, even for those who could hear the music, see the man, and read the letter, it was two fingers—held in the right way, of course.

The variety in kind of symbols referred to above, the possible radical nature of symbols, and the possible relation to philosophic monism or dualism indicate to me a deeper meaning to be uncovered in this kind of consideration of symbols than in a study of symbols of one kind, for example, of Christian symbols or heraldic symbols, or Indian symbols, etc. I add notes referring again to the radical nature of symbols and consequent philosophic implications.

The Radical Symbol. This term is used a number of times in the notes. It may be that as these notes on symbols have developed, the expression "radical symbol" has been the connecting element running through them.

Metaphor, aphorism, dialogue, epic, poem, picture, statue, algebraic equation, differential equation, etc., or significatives and representatives in the history of religion—to call these "symbols" may seem strange. If we want to be literal, they are compound symbols because they are constructed of many elemental symbols: the verbal statements are compounded of letters, the picture of strokes of a brush or pen, the statue of cuts of the chisel, mathematics of letters and other marks, significatives and representatives of water, stone, trees, altars, and later of images. However, the dictionary justifies the use of symbol for a large variety of things beyond an individual stroke, such as symbolic book, equation, argument, or a totality such as a state or a church.

For want of inspiration to find a better term I call these compounded representations simply "symbols." What then is meant by "radical symbol"? In a previous note I credit Wheelwright for suggesting to me this term (see *Heraclitus,* Philip Wheelwright). He is describing the special nature of the metaphor in the aphorisms of Heraclitus that distinguishes them as philosophical statements from other statements of similar grammatical construction. In the process he uses the expression "radically metaphoric" (p. 95, Atheneum edition). He notes for example that a "merely grammatical metaphor" is one that can be restated as a simile. "He is a pig" can be reduced to "He eats like a pig." However, he says that "God is reason" cannot be so reduced. We read:

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We seem to know both what God is and what reason is. We seem to have discovered a similarity between two independent ideas. This condition, he says, is adequately met in calling a man a pig, but not in the case of "God" and "reason."

Parenthetically, readers of this note might not wish to assert the metaphor "God is reason." But that does not at all take away from the illustration, because to say "God is Truth," or "God is Love," or "God is Wisdom Itself" would equally well serve the purpose. In fact they, as well as the example "God is reason," serve whether we believe in them or not. The atheist would probably grant that if one believes in God, "God is Truth" is a metaphor that cannot be reduced to a simile.

In these notes I use the expression "radical symbol" to identify the various formal means by which man has tried to communicate with man, and in the case of Revelation, the means by which God communicates with man. Heraclitus with his aphorisms, Plato in his Socratic dialogues, Dante in his epics, Giotto in his murals, Michelangelo in his sculptured figures, Newton and Schroedinger in their mathematical equations, are examples of the former, that is, man communicating with man. Their product is a radical symbol.

The significatives in the mind of man in the Most Ancient Church, which became the representatives with the Ancient Church, the written Word with the Hebrews, the Christians, and with the New Church, are examples of the latter, that is the means by which God communicates with man. And these means are examples of the radical symbol. They are irreducible to each other although there is a relation through correspondence—a subject important on its own account when dealing with symbols.

What are the properties of a radical symbol? These properties go beyond communication only. In the present and past sets of the notes I illustrated the following properties:

1. The radical symbol cannot be reduced. For example, the metaphor that cannot be reduced to a simile, the picture to words, mathematics to words or pictures—not even to numbers or geometric forms! The radical symbol expresses something that cannot be expressed in any other way. The radical nature of Giotto's murals remains even after the great majority of people learn to read the Bible. That is, the murals cannot be reduced to words; there is something in them beyond words.

2. The radical symbol extends knowledge. This extension is twofold—that is, an extension both within the universe of discourse, using the symbol itself, and also outside. Good painting breeds more good painting. But good painting also communicates something that stimulates one to think more deeply about the subject matter to which the painting is applied. So with good verbal discourses, good mathematics, etc.
3. The radical symbol has special timely historical significance. I will note later how significatives are attached in time especially to the Most Ancient Church, becoming representatives to the Ancient Church, and aphorisms, the dialogue, epic, etc., to philosophy in due course of time, and so also various forms of art and their special representations—in music such as the folk song, the chant, and fugue, the symphony, etc.—and finally in mathematics, *e.g.*, algebraic and differential equations.
4. Finally, the radical symbol has a great variety of uses. Note how Plato uses the dialogue to bring real persons to argue for and against sophism, Galileo uses imaginative figures in his dialogues to argue experimental science (he was horrified, no doubt, to hear that his enemies were whispering in the ear of his friend, the Pope, that Simplicio was the Pope), and Bishop Berkeley uses dialogue among individuals who stand for schools of thought (Philonous representing Berkeley or the idealist, and Hylas a materialist), as a means of exposition of his metaphysics. So also there is the great variety of other verbal forms, and a great variety in kind and in applications of other forms in art, and in mathematics.

Symbol as a Function of Time. The title of this note is itself a symbol. The meaning of "function" is a modern one—one that could not exist before the development of algebra which itself is a wonderful symbol.

How does the meaning of a symbol change with time? In Hugo Lj. Odhner's address, published in the July–Sept., 1965, NEW PHILOSOPHY, he refers to the place of representative truth—a "form of human communication—older than speech or words." There is a still older symbol than representations and that is significative truth.

In the present set of notes I will illustrate a modern symbol, the algebraic equation. Then I will go back in time—even before philosophy was known—and briefly discuss representative and significative symbols.

If significative and representative symbols are older than speech or words, in what do they consist? If algebraic equations are a distinctively new example of symbols, in what do algebraic equations consist?

Symbols, Monism or Pluralism. The symbol *per se* is of interest as a tool of communication, also perhaps as a means of extending knowledge, as will be explained in some other notes. But from its representative point of view it can represent only one aspect of its object. This is implied by the very existence of the many radical symbols.

But what is the nature of this aspect? Is it an extension of the object itself? Surely in a monistic universe it seems that it ought to be. In a pluralistic interpretation of the universe it seems that there might be a wide variety of possibilities. (When we get to positivism in these notes it will be useful to point out a possible out for monism through what is called reflection, *i.e.*, as in a mirror.)

The Radical Symbol and Reduction. In a note above I express the idea that when a method of expression (in these notes called "symbol") is used in a true manner it can do what no other symbol can do. True, something of what is represented in a picture or in a song, etc., can be expressed in words. But there seems to be something in a true picture or song that no words can express. In the case of a song, for example, what does the music add to the words? Can *this* be reduced to words too?

Comte, early in the nineteenth century, described an hierarchy of intellectual disciplines. To simplify by referring only to the two ends of his hierarchy: he considers religion as the oldest and obsolete, and social science as the newest and the crown of disciplines. At the same time Comte laid the basis for modern positivism.

But positivism receives more emphasis today from physical scientists than from social studies (sciences?) people. Positivism also is more concerned with reduction than with hierarchy. Historical events in thought at different times have raised followers of Religion, of Euclid, and Newton—perhaps even of Social Studies, and very likely the Vienna School in logical positivism and the Copenhagen School in their interpretation of the meaning of quantum mechanics, almost to the point where their discipline or at least their interpretation seemed to be the true way of academic thinking. These have each been challenged and in some cases the followers have come tumbling down from their high places. In some cases this fall is softened by a modification rather than a renunciation. I am not here talking about an acknowledged improvement in time, as for example modern physics over Galileo, but of an hierarchy such as that of Comte.

It would take us too far afield, and anyway the details are too many, to integrate examples into these notes and to discuss their modifications or renunciations. But I will cite three examples more or less by title. The first can be appreciated generally by anyone who reads about art in the daily newspapers. The second is directly related to reduction itself—specifically of mathematics to logic. And the third is a right-about-face in positivism.

1) The present reaction to "abstract art." It is becoming quite fashionable now to slip gradually away from this fad into what is called "figurative" art. Michelangelo, da Vinci, Delacroix, and even the French Impressionists studied *anatomy*. As abstract art now seems to hit a dead end we are treated to the soft kind of pulling back into *figurative* studies.

2) In mathematics, the "Introduction To The Second Edition" of Russell's *Principles of Mathematics.* Russell, being a logician and mathematician, changes his mind in an explicit manner. And he does so in the above reference in a number of ways: (a) He challenges the unique place of "p implies q" as a logical form of mathematics in his *Principles.* (b) He gives a lengthy modification of his views on what he calls "logical constants." (c) He criticizes the original "theory of types" as well as other things in his *Principles.*

3) The about-face in positivism (with special reference to the Oxford School). In 1936, A. J. Ayer published a book, *Language, Truth and Logic.* C. E. M. Joad in his *A Critique of Logical Positivism* (Chicago: The University of Chicago Press, 1950) uses Ayer's book as his source of material. According to Joad, "[Ayer's book] has in Oxford since the end of the war acquired almost the status of a philosophic Bible." (*Ibid.* p. 9)

Nevertheless, already by the time Joad was preparing his critique, a second edition had come out. Joad says of Ayer's early position: "Intolerance is chiefly shown in a simple refusal to discuss metaphysical questions." (*Ibid.* p. 11)

The influence of this refusal is all too well known, not only in scientific literature and thinking, but even in philosophy right up to 1966. The softening, referred to above, was explained by Joad:

> As the exponents of the doctrine have grown older, the doctrine itself has grown milder and Professor Ayer now tells us that it is only to *one* proper sense of the word "meaning" that the verification principle applies. (*Ibid.* p. 11)

Ayer had previously questioned whether metaphysics had any meaning. It now appears that this softening process has become more straightforward and Russell-like. In *The Antioch Review,* Winter 1965–1966, George R. Geiger in "Notes on Philosophy" observes that

> it is pretty clear that "Oxford philosophy," *i.e.,* that of linguistic analysis, has been moving into new and unorthodox fields—that is to say, into old and traditionally orthodox fields. (p. 569)

According to Geiger, past members of the group are now writing on topics that are hardly proper topics for positivists. But specifically with regard to Ayer, Geiger says:

> To go on: A. J. Ayer, once the prince of logical positivism, has recently been attacking the very foundations of linguistic analysis. (p. 596)

Since these notes on symbols had their initial stimulation in a remark made by Hugo Lj. Odhner on positivism, I intend before the topic is brought to a close to make some comments directly on positivism.

However, reduction seems now to be a substitute for hierarchy. Even the social studies were treated to a phase of this reduction when they were called social *sciences.* The effort of Russell and Whitehead to reduce mathematics to logic, present-day efforts to reduce physics to mathematics, not to mention the positivist effort to reduce the source of all knowledge to empiricism, all appear as examples.

Developments in mechanical and electrical instrumentation in recent years have tempted people to give support to reductionism in many ways. The digital computer has spurred efforts to put on input tapes things no one before these times thought could be represented by marks of any sort. The digital computer has challenged people to try to reduce thinking itself to a complex of "bits" of information—that is, simple yes and no answers in a complex array of Higginbottom electronic circuits. Can thinking itself be so reduced?

How about the senses? Can they not be reduced? Norbert Wiener, the father of cybernetics, in a lecture I heard him give several years ago, was discussing communication. The end result of that particular lecture was the suggestion that with modern technology applied to electronic miniaturization one could equip a person with a bat-like sensitivity system. The obvious use to a blind person cannot be debated, and "seeing-eye dogs" may join the old grey mare in antiquity. The land of "seeing" thus provides a perception of silhouettes and distances, and would be wonderful for a blind person—but would hardly provide a satisfactory reduction for one with normal vision who has learned to use that vision in a thousand ways that go beyond silhouette and distance perception.

All of these things, or even any one of them, can be used as a basis for arguing in favor of reduction of some kind. And each of these arguments is against the existence of the radical symbol. But I believe that radical symbols do exist, that they cannot be reduced, that they are essentially related to the nature of creation itself, and that they can be seen in their proper perspective in time by a proper study of their history.

Mathematics and Popular Reduction. Aside from the efforts, mentioned above, to reduce one discipline to another, there is another kind of reduction. This is the popular kind. It is not consciously realized by those who practice it. An excellent example is the way in which "mathematics" is used ambiguously to mean so many things.

This stems from ignorance. Logic, mathematics, and physics are all mathematical when their symbols of representation are regarded only superficially. But if logic, mathematics, and physics are each faithful representations of some aspect of nature, then each must be irreducible, that is, to use the language used elsewhere in these notes, they must each be radical symbols.

Logic deals with the forms of thought. Mathematics applies the forms of thought to real existents: number, identities expressed in various forms, equations of various kinds, and other relations involving order, transformations, etc. Physics applies the apparatus of mathematics to physical existents: distance, force, mass, etc.

These comments on logic, mathematics, and physics are not intended as definitions. Such definitions would be very serious business indeed and are beyond the scope of these notes. They are intended only to show a contrast among the three areas by using their content. (See *Definition a Symbol* below.)

Logic a Radical Symbol. The history of logic itself would be interesting to develop as an illustration of a representation that is not linguistic in the usual sense of that word. Of course, Aristotle's treatment of logic is linguistic. He begins his *Analytics* with an analysis of definitions and the simple declarative sentence involving only forms of the verb "to be."

But since Aristotle there have been those with a growing "dissatisfaction with ordinary language." Descartes used the expression "universal mathesis" for "a language which shall be the perfect instrument of analysis and demonstration." (Quotes are from *A Syntopicon of Great Books of the Western World* under "Language.") A more recent reaction on logic as it moves away from a linguistic form is that of Jevons (1835–1882). We read:

> The hopes to be realized by an algebra of logic find expression in Jevons' plan for a logical abacus which, like an adding machine or comptometer, would be a thinking machine able to solve all problems that can be put in suitable terms. (*A Syntopicon, ibid.* p. 945)

Perhaps most famous of modern efforts in symbolic logic is the three-volume work of Russell and Whitehead, *Principia Mathematica.* A standard contemporary introductory text to symbolic logic is *Mathematical Logic* by Quine.

But whether linguistic or symbolic, modern logic is something quite different from what is usually understood by "linguistic" or "algebraic." We find a statement illustrating this in Russell's "Introduction to the Second Edition" of his *Principles of Mathematics.*

He is discussing what he considers to be a necessary modification of his use of "classes" in the first edition:

> Seeing that cardinal numbers had been defined [*i.e.*, in the first edition] as classes of classes, they also became "merely symbolic or linguistic conveniences." Thus, for example, the proposition "$1 + 1 = 2$," somewhat simplified, becomes the following: "Form the propositional function 'a is not b, and whatever x may be, x *is* a γ is always equivalent to x *is* a *or* x *is* b'; form also the propositional function 'a is a γ and, whatever x may be, x *is*

> *a γ but is not a* is always equivalent to *x is b.*' Then, whatever γ may be, the assertion that one of these propositional functions is not always false (for different values of *a* and *b*) is equivalent to the assertion that the other is not always false." Here the numbers 1 and 2 have entirely disappeared, and a similar analysis can be applied to any arithmetical proposition. (cf. p. x)

I believe that the above quotation is enough to indicate that neither the language nor the arithmetic used there makes for the average reader the sense Russell intends. For a novitiate who tries to translate his remarks, using only a dictionary and an arithmetic book, will hardly feel satisfied with Russell's words "somewhat simplified."

The reader should rest assured, however, that the quotation is not silly—it may be technically right or wrong, but it is not silly. The reader can also be assured that Russell believed in what he was doing, that is, in trying to reduce mathematics to logic. And after Russell has subjected his own work to critical analysis, that is substantially what he is honestly left with—namely, belief. (If the reader is interested in the arithmetic example given in the quotation, its relation to "The Principles of Mathematics," and to positivism, he will find a discussion by William Barrett in the "Introduction" to *Philosophy in the Twentieth Century.* Vol. 3.)

However, directly related to our topic of reduction, Russell says:

> The fundamental thesis of the following pages, that mathematics and logic are identical, is one which I have never since seen any reason to modify. (p. iii)

After discussing corrections with respect to the original treatment in *Principles* and giving critiques of opposing theories, he says,

> Broadly speaking, I still think this book is in the right where it disagrees with what had been previously held, but where it agrees with older theories it is apt to be wrong. (p. xiv)

I leave it to others to determine what this means and the basis for belief in it. At any rate we cannot ignore the possible relation to our broader philosophical concern in these notes, viz. monism and pluralism, for Russell concludes the "Introduction" with the sentence:

> How far it is possible to go in the direction of nominalism remains, to my mind, an unsolved question, but one which, whether completely soluble or not, can only be adequately investigated by means of mathematical logic. (p. xiv)

I hope some time in my notes to come back to Russell's "Nominalism" by way of his "neutral matters." On the other hand I personally do not believe in his kind of reduction. And it is in an effort to support my belief in the impossibility of reduction that I am writing these notes.

If one believes with Russell, then it is his job not only to show that reduction is possible but also to show that epistemology and ontology resulting from such a reduction lead to a monistic universe. Reduction is a difficult job. Most modern philosophers and practically all positivists are engaged in some aspect of this task.

If one, with me, does not believe in reduction, then there are a number of possibilities open to us. One of these is a belief in a pluralistic universe—and creation!—and if one goes this far he will find himself in the minority. There are relatively few competent philosophers engaged in a program to support this belief.

Pluralism of course has its problems, as does monism. "The scandal" of the mind-body problem of Descartes is usually enough to dissuade people from investigating the possibilities of pluralism.

"Universal Mathesis." Swedenborg has a section in *The Rational Psychology* entitled, "A Universal Mathesis, or a Mathematical Philosophy of Universals." In it he attributes to Locke the descriptive ideas leading to the idea of a "universal science."

In *The Economy of the Animal Kingdom* he says:

> Wherefore a mathematical philosophy of universals must be invented, which, by characteristic marks and letters, in their general form not very unlike the algebraic analysis of infinites, may be capable of expressing those things which are inexpressible by ordinary language. (1 Econ. 651)

Swedenborg does not seem to have received the idea of "universal mathesis" directly from Descartes. Aside from the reference to Locke in *The Rational Psychology,* he quotes in the *Economy* number above from Wolff who uses the term "universal mathematics." Also in *A Philosopher's Note Book* (p. 239) Swedenborg quotes from a letter by Leibnitz wherein the term "universal algebra" is used.

However, the development of such a "science," "algebra," or what have you, had to await more recent times, and is associated with the names of Boole (1815–1864), Venn (1834–1923), Russell (1872–), Whitehead (1861–1947), etc.

While Swedenborg was writing *The Rational Psychology* he seemed to foresee some difficulties in the development of the universal mathesis. History appears to bear this out as indicated, *e.g.*, in Russell's "The Introduction" referred to in these notes elsewhere. (See Swedenborg's remarks concerning anticipated difficulties *R. Ps.* 567.)

However, Swedenborg's interest at that time was already broader than natural science or even rational psychology. He was already interested in a pluralism that involves spiritual and natural things. He says:

> . . . . I have desired to propose a certain *Key of Natural and Spiritual Mysteries by the way of Correspondences and Representations,* which more directly and certainly leads us into hidden truths; and upon this doctrine, since it is as yet unknown to the world, I ought to dwell at somewhat greater length. (*R. Ps.* 567)

An effort along this line is given in *Hieroglyphic Key to Natural and Spiritual Mysteries by way of Representations and Correspondences.*

The broader interest of Swedenborg, at this time or even earlier, is indicated by the numerous quotations he includes in *A Philosopher's Note Book* under the headings, "Type, Representation, Harmony, Correspondence."

Items published earlier in NEW PHILOSOPHY that might interest the reader are:

Olds, C. L. "Translation of Swedenborg's Characteristic and Mathematical Philosophy of Universals." April 1903 (This work is also available in the fascicle *Scientific and Philosophical Treatises,* Part II)

Iungerich, E. E. "Is a Universal Mathesis one with the Science of Correspondences?" January 1931

Odhner, P. N. "The Universal Mathesis." January 1932

Also Alfred Acton has discussed Swedenborg's interest in the Mathematical Philosophy of Universals, Vol. III, pp. 553–555, *Notes on Life of Swedenborg,* unpublished, A.N.C. Library.

Definition a Symbol. The very word "definition" is itself a symbol. To deny its radical character is to deny the representation of knowledge in any form—any kind of knowledge. Even the grunt of an animal means something. But on a more human plane it was the very challenge of the process of definition itself in degraded sophism that Socrates was speaking out against. It is not the dialogue that is the rock-bottom method in Plato, it is how to define. It is not virtue, or the state, or education that is the elemental topic of the *Dialogues* and *The Republic;* it is knowledge itself, reconstructed through acceptable definitions. It was a crying out against Sophism in these writings of Plato that made them the new beginning of philosophy after Sophism had done its part with earlier pre-Socratic thought.

Strictly speaking "the serious business" (see above *Mathematics and Popular Reduction*) of defining the disciplines logic, mathematics, and physics—or language, or art, etc.—*is* a proper subject of these notes if we are to attain a deep feeling for their radical or irreducible character. My point in the previous note, however, is that to define is too much for these notes—or for me. Maxime Bôcher tried to do it for mathematics many years ago. When I read B. Russell I seldom know when he is talking of logic or of mathematics. Bôcher and Russell are far better mathematicians than I.

Certainly definition is not something accomplished with mere words. *Webster's Unabridged Dictionary* cannot do it with "Laocoön," nor again with "natural logarithm."

The dictionary does not pretend to be a work of art, but in order to express Laocoön it produces a line drawing of an object now in the Vatican and "found in 1506 in the Esquiline Hill in Rome." Whether or not El Greco was inspired by this particular object, I do not know, but anyway his definition of Laocoön hangs in the National Art Gallery in Washington. What was once before him as blank canvas is now almost entirely filled with flesh, both human and serpent, leaving negligible room for the famous wooden horse and not very much more for the city of Troy. According to the line drawing in the dictionary the art object in the Vatican seems to be related to the story of Laocoön as it comes down to us. For I seem to see only male figures when they are human and can be disentangled from the serpents. But in El Greco's version there appears a female—human, that is; I would not recognize a female serpent. I am informed she appeared in a recent cleaning of the painting. And so if El Greco was inspired by what the Esquiline Hill yielded in 1506, 42 years before his birth, he apparently was not initially bound literally in the under painting of his execution of his definition.

As for the dictionary definition of "natural logarithm" we find the following

$$\log_e x = \int_1^x \frac{dt}{t}.$$

I will return to this in a future set of the notes.

But let the above be evidence that definition itself is a radical symbol and cannot be reduced to words.

(With reference to Laocoön see The *Æneid* of Virgil, Book II. Concerning the statuary group in the Vatican see *Treasures Of The Vatican,* Calvesi [Skira]. Concerning natural logarithms see, *e.g.,* *Calculus and Analytic Geometry,* Thomas.)

The serious business of defining is related to the radical or irreducible character of symbols. But in defining non-linguistic symbols—logic, mathematics, physics, art, music, etc.—more than words are needed. Examples as in the cases of Laocoön and natural logarithms help. But only through living with these things and in their use can we make contact with the truths proper to the symbols of which they are examples. I even wonder about the linguistic forms. Do we know "aphorism," "dialogue," "epic" from the dictionary?

Mathematical Objects. Every discipline accepts the existence of certain objects. No one I know would choose to deny the existence of the natural numbers

$$1, 2, 3, \ldots$$

as objects existing for mathematics.

Nevertheless if one asks, "Upon what bases are these accepted?" one is due for astonishing answers. There is no unique one. And every reason that has been given has had its proponents and its opponents.

The problem involved is two-fold. We are at the rock-bottom of fundamentals of mathematics. And what constitutes, and what is the meaning of, rock-bottom concepts always loom as difficult problems in any logical subject.

In order to talk about the kind of mathematics I will discuss in these notes, it is convenient simply to accept the existence of the natural numbers and assume that the reader will have a sufficient intellectual reaction to them to follow the intent of the discussion, whether he be a logician or mathematician, whether he be an advanced mathematician or one not versed in mathematics, yet otherwise reasonably well educated, whether he be a Pythagoras, a Euclid, a Diophantus, a Gauss, a Kummer, a Dedekind, a Cantor, a Russell, a Brouwer, or a Weyl. (The mathematician who is interested in the history of mathematics will recognize the names I give as those of representative men who have had a notable influence on possible meanings of number, and in consequence each has his followers. The literature on resulting discussions from these influences is vast. The interested reader is referred to the following for an introduction: B. Russell, "Introduction to the Second Edition" of his *Principles of Mathematics*, E. V. Huntingdon, *The Continuum, And Other Types of Serial Order*, Fraenkel, *Abstract Set Theory*. The last reference gives a list of related papers filling 126 pages, about 18 references per page. This gives some idea of the attention that has been given this question, and also gives some idea of what I might be skipping in order to get "down to the brass tacks" of talking about the number system.)

The Natural Numbers. I have said that I wish to accept the following series of natural numbers without further reduction:

$$1, 2, 3, \ldots .$$

My readers who have not made a study of the principles of mathematics or of logic will hardly sympathize with my emphasis with regard to this point. Nevertheless there is much in the literature against such an acceptance. Russell's "somewhat simplified" representation of $1 + 1 = 2$ given in another note is a case in point. It can be noted that one of the characteristics of that representation is the effort to avoid using an expression which in any way refers to the cardinal numbers.

Again in his *The Analysis of Matter* (p. 3) Russell defines infinite series, as he says, "without mentioning integers."

To go to another author, in *The Continuum* (p. 3) E. V. Huntingdon says:

> It will be noticed that while the usual treatment of the continuum in mathematical text-books begins with a discussion of the system of real numbers, the present theory is based solely on a set of postulates the statement of which is entirely independent of numerical concepts.

More in the spirit of mathematics than of logic, in a well-known text on modern algebra the authors say:

Instead of trying to define what the integers are, we shall start by assuming that these integers, whatever they are, must satisfy basic algebraic laws. (*A Survey of Modern Algebra*, Birkhoff and MacLane, p. 1)

And in a somewhat more naive manner the authors of a more popular book on mathematics say:

The integers have gradually lost their association with superstition and mysticism, but their interest for mathematicians has never waned. (*What is Mathematics?* Courant and Robbins, p. 21)

Earlier still the numbers were important because of their representations and still earlier for their significations. At any rate they have been around for a long time and so it seems reasonable to accept their reality on other than logical grounds—grounds that seem to date from, at the earliest, not more than maybe 100 years ago.

Mathematics as a Radical Symbol. That mathematics is a radical symbol is superficially evident in that mathematicians as thinkers are often set apart from others. This is usually not done as much by mathematicians as by others. Nevertheless mathematical thought must represent a significant aspect of nature and of thought itself because of the considerable influence of mathematically prepared thinkers in philosophy.

In the next few notes I will endeavor to illustrate how mathematics satisfies the notion of a radical symbol in that mathematics cannot be reduced to words, to number, to form (*i.e.*, geometric form); it extends knowledge, that is, of itself it expands and also it extends knowledge in other fields; it has a special timely significance both as to itself and as to its scholars; and finally there is a great variety of mathematical symbols (in the sense of these notes, not only literally) and also there is a great variety of applications of mathematics.

In order to develop this theme I am forced to refer explicitly to some mathematical representations. The reader who is one of those inclined to put mathematics in an intellectual universe apart from his own will please bear with me. I am going to refer to the mathematical representations by title only. I am going to refer to numbers and how they came to be discovered, without asking that the continuum or fundamentals of logic be understood; I am going to refer to algebraic equations without asking the reader to find the solutions—those required are all given.

Mathematical Symbols. The best-known mathematical symbols are the series of natural numbers:

$$1, 2, 3, \ldots.$$

The Romans had symbols for these but mathematical work with their symbols is severely restricted from the modern point of view. One might think that the reason for this is the cumbersome nature of their numeral representation, but this is not entirely the case. The fundamental reason is that they had no symbol for zero.

The reader will immediately recall from his school-day arithmetic how pairs of integers can, by division, form fractions, and how, using another symbol, the decimal point, the fractions can be given an alternate representation in the decimal form.

Of course the operations addition, subtraction, multiplication, and division each have their symbols. But for the purpose of this note what we are especially interested in is the property that symbols have to extend our knowledge. There are many knowledges where words are inadequate for their expression. Mathematics is rich with symbols where the concept came before the word even though words have been later coined to represent the concept. "Zero" and "negative numbers" are two examples.

The symbols that represent the irrational numbers are another case. Consider for example:

$$\sqrt{2}$$

It is true that this number can be represented with increasing accuracy by the series 1.4; 1.41; 1.414; 1.4142; etc. But no series of approximations, however long, will ever represent $\sqrt{2}$. Somehow $\sqrt{2}$ says more than such a series.

$$\pi \text{ and } e.$$

These are but two examples of what are called the transcendental numbers.

Relation of Numbers to Mathematical Equations. Counting, at least up to two, involves perhaps the most primitive beginning of mathematics. And yet counting is itself not mathematics; it is an art. (See, however, for a different opinion *Number the Language of Science,* Dantzig.) Mathematics does not begin until we have a symbol for an abstract number, thus for example:

$$1 \text{ and } 2.$$

The symbol that stands for what is in common with a man and a woman, a pair of trees, or of apples, etc., is 2. Logicians and mathematicians alike have had trouble defining "number." The problem begins with defining "one."

In discussing 2 above I was able to appeal to something that is common to a "pair" of these, a "pair" of those, etc. Shifting from "two" to a "pair," although superficially satisfactory, is hardly giving a definition. An appreciation of the difficulty of the problem is emphasized when we try to define 1 in the manner in which we "defined" 2 above, that is:

We point out a number of individuals: a man, a tree, an apple, etc. What is it that these individual objects have in common? We might note something of their "individuality," or their "aloneness" that seems somehow related to their "oneness." But we might equally well see that each is a "different" form of substance, or a "representative" of a group, etc., notions that seem fairly remote from 1.

The implication above might lead us to suspect that it is easier to define 2 than it is to define 1. Grant this. Then how to define 1 in terms of 2? From our advanced state this can be done if we could somehow define 1/2. Then 1 is 1/2 of 2. (An amusing commentary on defining "number" is given in *Science and Method,* Henri Poincaré, in the chapter "The Latest Efforts of the Logicians.")

There is a much easier way of going about this and that is to accept the series of natural numbers

$$1, 2, 3, \ldots$$

without definition. I am by-passing an enormous literature on the foundations of "definition" itself when applied to "number." But at any rate the truth is that the series of natural numbers is abstract. That is, their existence depends upon something other than concrete objects. And no amount of dealing with concrete objects will clarify the nature of the notion of number. We are in a new universe of discourse. Number is another example of a radical symbol. Its notions are peculiar to itself, and once we understand this we have a marvelous world of existents: the mathematics which depend upon number. (It should be mentioned parenthetically that there are other kinds of mathematics depending on other symbols.)

Having accepted the sequence of natural numbers where do we go from here? Euclid knew that the integers as they are called are inadequate to take care of geometry. For example, let a point on a line be named 0 and a different point be named 1. By a well-known construction the line 0 to 1 can be bisected. What is the name of the point of bisection? A new number is required that does not appear in the sequence of numbers

$$1, 2, 3, \ldots .$$

We call it 1/2. And so it goes for all the points that result from bisections, trisections, etc. Hence we have the rational fractions.

Furthermore, let 0 be the name of a point on a line and 1 another point on the line to the right of 0. What about the point that is to the left of 0 a distance equal to the distance from 0 to 1? Since Descartes we call this point -1. Hence we have the negative numbers. (My historical reference here is somewhat inaccurate, but I follow the common habit that since Descartes is the inventor of analytic geometry we credit him with what we know today about this branch of mathematics. Also above, in the case of Euclid, lines had length but it was not correct to assign numerical names to points in his time.)

Furthermore, let us have an isosceles right triangle each of whose sides that form the right angle is equal to 1 unit in length, that is, the same distance as from 0 to 1 above. What is the length of the hypotenuse? According to the Pythagorean Theorem it is $\sqrt{2}$. But this number does not appear in any of the symbols so far mentioned. Euclid proved that the $\sqrt{2}$ cannot be represented as a rational fraction. (See, *e.g.,* Heath edition, Vol. I, p. 351)

The temptation is to extend geometrical constructions by various means and arrive at still other numbers. But we run into difficulties. Geometry itself demands the existence of the ratio of the circumference of a circle to its diameter. This number is called "pi" and is represented by π. It is known that π is a number between 3 and 4, and so there is located on our line mentioned above, somewhere between the point called 3 and the point called 4, a point we name π (out to a few places this is 3.14159). But how does one find that point by geometrical construction? From independent sources we know of a vast collection of other numbers that exist but are not locatable on our line by geometrical constructions. Take as a prominent example e, the natural logarithmic base.

It appears that "number" is a concept much more abstract than we suspected when trying to define 2. For in that abstraction we seemed to want to abstract only from physical objects such as men, trees, apples, etc. Now we find a need to abstract from geometry itself—i.e., a part of mathematics! "Number" is truly a radical symbol, for its definition cannot be reduced and also its use extends our knowledge.

One convenient way of pursuing the abstraction of numbers is through the use of algebraic equations, to be specific, integral rational algebraic equations in one unknown. They are integral because all powers are whole numbers. They are rational because the coefficients are all rational.

A very simple illustration is

$$x - 1 = 0.$$

In the language of mathematics this is a question. The question asked is: What value of x satisfies the statement, *i.e.*, from what can one be subtracted to give zero? The answer is 1.

Another example is

$$2x - 1 = 0.$$

The answer to this question is that x stands for 1/2.

Of course it is admitted freely that in the very definition of such equations the existence of the sequence of natural numbers, negative and positive, and the existence of rational fractions are accepted. But these algebraic equations represent new symbols which extend our knowledge about number in a natural manner. For example, as just illustrated, the rational fractions can be defined by the equation

$$ax + b = 0$$

where a and b are each one of the integers, positive or negative. Other numbers will be defined in the next note.

Mathematical Equations and Algebraic Numbers. Mathematical equations are representations requiring three or more symbols. There is a vast variety of mathematical equations, but those that appear in high school algebra books can be classified under two headings. There is the kind of equation that is a statement, or, if you please, a declarative sentence. This is an example:

$$(a + b)^2 = a^2 + 2ab + b^2.$$

Notice that this statement is true for any pair of the numbers mentioned in the previous note. Such an equation is called an "identity."

The equation given in the note above, *i.e.*,

$$2x - 1 = 0$$

is different. It is not a declarative sentence; as noted there it is a question. The question is what number, multiplied by two and decreased by one, gives zero? No natural number can be the answer. And so to find an answer we must invent a new kind of number 1/2, which is an example of a rational fraction. As another example,

$$3x - 7 = 0$$

defines the rational fraction 7/3.

Another kind of algebraic equation is called quadratic. An example is

$$x^2 - 5x + 6 = 0.$$

The question asked by this equation is, What number multiplied by itself and then decreased by five times itself and then increased by six gives zero? One can easily see that either $+2$ or $+3$ will be a correct answer.

If we write a somewhat simpler quadratic equation,

$$x^2 - 4 = 0$$

we see that both $+2$ and -2 are answers.

However, let us write

$$x^2 - 2 = 0.$$

We find there is no answer to this question if we are limited to natural numbers (plus or minus) and rational fractions.

To give answers we invent new numbers, viz., $+\sqrt{2}$ and $-\sqrt{2}$. It was seen in a previous note that $+\sqrt{2}$ was also required on geometrical grounds. This indicates that there is a relation between some things in geometry and some things in algebra. The discipline known as analytic geometry extends this correspondence.

For the moment pursue a little further the radical nature of some equations similar to

$$x^2 - 2 = 0.$$

We note in this particular case the evolution of a new number, which also happens to represent the length of a certain line in geometry. Is this reduction or close connection between geometry and algebra possible throughout algebra and geometry? Reflect for example upon

$$x^5 - 2 = 0,$$
$$x^{131} - 2 = 0, \text{ etc.}$$

The numbers defined by these equations are called "algebraic numbers." And algebraic equations involving orders higher than two generally cannot be represented by geometric lines.

To illustrate still another kind of number that is defined by algebraic equations, write

$$x^2 + 4 = 0.$$

A careful examination shows that none of the symbols so far mentioned as representing numbers provides an answer to this question. What number when multiplied by itself and increased by 4 gives zero? In order to provide an answer to this question it is necessary to invent new numbers—the so-called imaginary numbers. The two answers to the above question are then $+2i$ and $-2i$, where i^2 is defined as -1.

(As with $\sqrt{2}$ there are also corresponding geometrical representations for imaginary numbers. And so the mathematician of course will realize that it would be improper of me to isolate geometry and algebra completely so early. As we are fully aware of the relation of Demoivre's Theorem on complex numbers to the D'Argand Diagram, the plotting of the nth roots of unity, etc. However, even this new addition to geometrical representations has limitations in representing generally numbers that are solutions of algebraic equations.)

These notes do not have for their purpose a systematic exposition of numbers. The purpose is only to illustrate the use of symbol in mathematics to evolve new numbers. Properly conceived symbols lead to a rich development of knowledge that goes beyond the horizon of ordinary language, just as does the invention of the symbol made with the paint brush and properly selected pigments or proper cuts with a chisel, etc.

Many other symbols of considerable use will readily come to the reader familiar with mathematics, for example to extend the concept of number he will recall complex numbers, the concept of infinity (*i.e.*, ∞) and the transfinite numbers of Cantor. See the following respectively for treatment of numbers: *What is Mathematics?* Courant and Robbins, and for a more complete treatment, *Abstract Set Theory*, Fraenkel.

Mathematical Symbols and History. There are various forms by which words are arranged in order to communicate—prose, poetry, exclamations, etc.

As already mentioned, some of these have become especially important in the history of thought in that they have lent themselves to a special self-conscious representation with regard to reality. These verbal forms are better known by more people than the variety of mathematical forms. And yet it is by means of mathematical forms that many minds, well known in the history of philosophy, were trained. The effect of this training is evident in their philosophy.

To mention a few examples: Plato in the dialogues concerning ideas; Aquinas in trying to bring reason to bear on problems of faith; Scotus in separating the things of faith from those of reason; Descartes in trying to establish in philosophy a certainty that is the same as illustrated by geometry; Kant in his treatment of his question "Is metaphysics possible?"; philosophers who, after Newton, use mechanics to support materialism; philosophers since 1926 who use quantum mechanics to challenge determinism. Sometimes, as with Aquinas and Descartes, it is more the method of mathematics that comes to the fore; other times, as with Plato and philosophy since 1926, it is more explicitly the symbol. But the method itself, once developed, as for example by Euclid, nevertheless rests upon a special mathematical form.

Significative vs. Representative Symbols. In the history of the churches an object which stands as a symbol can do so in two very different ways. A series of numbers in the *Arcana Coelestia* describes this (AC 665, 920, 1321, 1409).

According to this series the most ancient man felt within himself the spiritual or heavenly signification of everything about him. Such men constitute what the Writings call the Most Ancient Church. This "internal" feeling in himself was an actual feeling.

> And so it was with the man of the Most Ancient Church: whatever he saw with his eyes was heavenly to him; and thus with him everything seemed to be alive. And this shows the character of his Divine worship, that it was internal, and by no means external. (AC 920:2)

This feeling *within* one's self brought an immediate conjunction with the things of heaven. This sensation within one's self is what the Writings call "perception." This meaning of the term "perception" is to be contrasted with the modern one where "perception" has reference to the external senses.

As the Most Ancient Church declined, man lost his internal perception, but in the Divine Providence the symbols in evidence outside man were nevertheless given a representation which corresponded to the earlier perceptions. These significatives were collected together by those who were called "Cain" and "Enoch." Those who follow are called by the Writings the Ancient Church. Because of the close correspondence between what was internally sensed by those of the Most Ancient Church and the collections of the Ancient Church, it is said "and consequently their writings also were of the same nature." (AC 920:4)

But the things which were now representative symbols were outside man and could only be perceived in the modern sense, that is by the external senses, and the Writings say:

And as in these representatives they admired, and seemed to themselves even to behold, what is Divine and heavenly, and also because of the antiquity of the same, their worship from things like these was begun and was permitted, and this was the origin of their worship upon mountains and in groves in the midst of trees, and also of their pillars or statues in the open air. . . . (AC 920:4)

Note that the worship was "*from* things like these," not "*of* things like this" (italics mine). Nevertheless after further decline there were

at last the altars and burnt-offerings, which afterwards became the principal things of all worship. (AC 920:4)

As long as the symbols were significative, one learned from them internal things, and from them thought of spiritual and celestial things.

But when this knowledge began to perish, so that they did not know that such things were signified, and began to regard the terrestrial and worldly things as holy, and to worship them, with no thought of their signification, the same things were then made representative. (AC 1409:2)

Radical Symbols Illustrated by Significatives and Representatives. The radical symbol as used in these notes has four properties: an irreducible character, extends knowledge, timely historical significance, and a great variety of uses.

It may seem to some incongruous to discuss algebraic equations and significative and representative symbols in the same series of notes. Yet it seems to me that each possesses the four properties listed. To recognize these properties seems to produce a perspective that places each in its proper universe of discourse, places each in an epistemological and historical perspective, and also gives us a framework in which we can understand the use in each case. It seems important to me to be able to identify within each of the important forms of communication, whether man-made or in revelation, the distinct properties that give each its unique character (why for example do we have all these forms of expression: verbal, art, math, etc.?). It seems important that each of the important symbols is an important source of knowledge. Does not each have a peculiar timely historical meaning? It seems that each is not trivial; it has a rich variety of uses.

Take one aspect: the historical, how to understand the relationship of the Most Ancient and Ancient Churches to our own time? Consider one single representative idea, "the flood." Is this purely representative, or does it have a physical reality in time? Is the receptacle of life we call man today distinctly different than in those former times? If so, then the very nature of significatives and representatives as they appeared to those ancients must be essentially different from what we today understand by them.

Swinging to the other end of the spectrum of symbols, consider differential equations. Differential equations, so different in kind from significatives and representatives, are also specially tied to a period of history. To illustrate this: In our day there is a tendency among some in physics to identify natural law with differential equations! This conception, whether true or false, would have been impossible before Newton's time—or at the very earliest Galileo's.

But returning to significatives and representatives, it seems to me that it helps to separate out their meaning from that of other symbols, such as the grammatical metaphor and other verbal or mathematical or artistic forms. It seems to me that it helps us to approach truth by investigating the properties of the radical symbol:

1. The irreduction aspect. Revelation, whether Ancient, Christian, or that through Swedenborg, cannot be reduced to philosophy or science.
2. Extension of knowledge aspect. Not only do we extend our knowledge by learning the scientifics of revelation themselves, but once this extension begins there can be an extension of knowledge outside—*i.e.*, to spiritual, to moral, and even practical life.

3. The timely, historical aspect. The symbols, whether immediate as with the Most Ancients or mediate, whether carved on tablets of stone or written in Hebrew, Greek, or Latin, have distinctly timely significances. What emphasizes this more explicitly than the expression *Nunc Licet* (TCR 508)?
4. Finally the aspect of rich variety. The significatives or the representatives with any individual that are essential for his own regeneration may be few and often simple. Yet the sum of all these taken collectively, not only in a single age but taken collectively throughout the history of the churches, must be enormous in variety.

Correspondence. This is a term from the Writings of Emanuel Swedenborg. Its meaning is essential to the distinctive position of the New Church. If the distinctiveness is genuine then the New Church itself cannot be reduced to Christianity as ordinarily understood. It is Christian in that it believes in the reality of Jesus Christ and accepts the stories about Him given in the Gospels. But with these things accepted the agreement with the Christian Church and its various derivatives ceases.

The distinctiveness is illustrated through the term "correspondence." There are two worlds: a spiritual world and a natural world. There is a correspondence between the things of the two worlds: the spiritual sun and the natural sun, for example.

The distinctiveness is illustrated again within things on this earth. The books called The Word or Revelation are written in verbal symbols which make ordinary sense. But this sense is called the natural sense. Behind that sense is what is called the spiritual sense. There is a correspondence between the things of the spiritual sense and those of the natural sense. See for example *The Arcana Coelestia,* concerning the book of *Genesis.*

I suggest there is a relation between the terms "correspondence" and "irreducible or radical" as applied to other things in these notes. One of the best illustrations for me is in mathematics and physics. Much of what is physics can be represented by mathematics. And so there is a correspondence between the physical objects of study and mathematical objects. But physics cannot be reduced to mathematics.

Again consider Laocoön painted by El Greco. I have no doubt that it could be fairly well stored on magnetic tape. The individual brush strokes can be given a sequence arrangement which places them in one-to-one correspondence with the sequence of natural numbers (Dimension 1). The center of gravity of each stroke can be located on the canvas using Cartesian rectangular coordinates (Dimensions 2 and 3). For underpainting or overpainting another dimension can be added (Dimension 4). The direction of the stroke can be given (Dimension 5). Its length (Dimension 6). Its color (Dimension 7), etc., etc. Even the glazes can be described in this manner. How many dimensions will be required I do not know. But since El Greco painted the picture and he only lived from 1548(?) to 1614 he could only have put a finite number of "bits" of information on the canvas. I doubt if the process described would be nearly as difficult as programming a rocket flight to the moon. Once done, Greco's Laocoön would be on tape. (Someone might suggest T.V. as being simpler. But I say that would be a quick and dirty way to do it. My way would be more precise. If we want to have a simpler case use Pissaro instead of El Greco. But, so far as I know, Laocoön was not done by a pointilist.)

Now what is the relation between the information on the tape and that on the canvas hanging in the National Museum? Is there not a one-to-one correspondence between the pieces of information on each? But I hold the correspondence is the only real thing—reducibility is not. Each is a radical symbol. It is interesting to note that the information on the tape and that on the canvas are entirely different as to time. One can look at the picture and see the whole picture—if back far enough. But to gather the information from the tape takes time. The tape must be scanned or, as they say, "played back." The picture can be viewed instantly, albeit in time. But the tape must be viewed atomistically one bit after another. Here is a correspondence between something that can be presented instantly and that which takes time. (I am talking here about reception not appreciation.)

If correspondence means anything at all, it seems to me, it must be applied to two radical symbols—two where one cannot be reduced to the other. When applied to things in its distinctive New Church way it is applied to two things which differ by discrete degrees. But saying this only opens up new questions and new possibilities.

E. F. A.