PROBABILITY: A VIEW OF NATURE

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"The Divine Providence works...invisibly and incomprehensibly in order that man may in freedom ascribe an event either to providence or chance; for if providence acted visibly and comprehensibly, there would be danger of man's believing, from what he sees and comprehends, that it is of providence, and afterward changing to the contrary. Thus...truth would be profaned..." AC 5508(2).

Introduction

Much of the complexity and strength of science, especially physical science, is due to the pervasive use of mathematics, a tool for modeling phenomena and then projecting future developments and behavior. This kind of picture of nature, originating about the time Newton,[1] was a departure from the philosophical or rationalist approach of a Descartes or indeed, Swedenborg, and had, I believe, two consequences. First, the mathematization of science eventually gave man a capability of predicting and controlling physical events which seems almost magical to the uninitiated. Second, the emphasis on quantification seemed to have caused a localization of viewpoint, an emphasis on specific problems which are sufficiently confined in scope that there is a reasonable expectation of solution. The idea is that if enough small, local problems are solved the global solution or 'grand theory' will evolve. An example of this is the many pieces of physics and chemistry which would be necessary for a complete and satisfactory theory of pre-biotic evolution. In Swedenborgian terms this amounts to the use of what are primarily effects being collected to determine causes, an approach which must be used with caution. Yet having said this, the impression is not to be given that science is somehow invalid or corrupt because it is focused. On the contrary; the models which the human mind lays upon nature are reflections of both nature and the mind itself. These reflections have value and may complement and infill the

[1] While the wide application of this mode of thought is properly associated with modern (post-Newtonian) science, nevertheless it should be noted that Archimedes many centuries earlier had employed mathematics to express ideas concerning some static and dynamic aspects of nature, as well as for predictive purposes. See, for example, Joseph E. Brown, "The Science of Weights" in *Science in the Middle Ages*, University of Chicago Press, 1978. Ed.

general structure of a religious and philosophic view of reality.

In the centuries succeeding Newton two distinct types of models have developed: deterministic and probabilistic. The deterministic model is one in which the relevant initial conditions are assumed to be precisely known together with the operative forces. The effect of the forces on the system is precisely represented by a set of equations. These equations may then be solved (at least in principle) to yield exact values of the desired physical parameters such as position, momentum, electric field strength, and so forth. The probabilistic model is also quite mathematical but differs from the deterministic model by assuming some degree of ignorance or at least a lack of precision in the state of knowledge about the system or its operative forces. In this paper we propose to examine the role of the probabilistic model as a representation of reality. Does it provide an accurate reflection of the physical world or is it simply man's way of diminishing the problem to the point of tractability?

The plan of this essay is as follows. First, a brief summary of the development of deterministic and then probabilistic models up to the end of the 19th century will be given. The intent is to describe the introduction of probability as a pragmatic necessity and to give some sense of the vast amount of scientific territory that is treated in this manner. Second, we turn to the question, from a scientific view, as to whether probability is the only mode of description for microscopic matter. That is, we describe some of the more explicitly probabilistic results of quantum theory. In this discussion some elementary distinctions will be made between the words 'causal[7] and 'deterministic[7] as they relate to scientific theories. Finally, we discuss the question of 'chance-in-principle[7] in the material world. Is the physical world, in its deepest parts, accurately and maximally reflected by a probabilistic model?

With this format in mind we begin with a look at the past.

Historical Review

1. *Deterministic models*

Perhaps the archetypical model of physical reality is the mathematical expression of Newton's second law:

$$\frac{dp}{dt} = F(r,t)$$

where the left side describes the rate of change of a particle's momentum and the right side indicates that the change of state is the result of the applied force. Provided the initial velocity and position of the particle are known, together with the explicit form of the force law, $F(r,t)$, all subsequent dynamical characteristics of the particle may be known precisely. Combined with Newton's law of gravity, the above equation may be used to predict the orbit of a planet about the sun fairly well. Certainly, predictions of planetary positions had been made prior to Newton's time, but the utter simplicity of the Newtonian 'equation of motion' approach must have been overwhelming, and it is reasonable to view Newton's work as the beginning of the mathematical modeling paradigm.

Despite certain philosophic problems such as the actual nature of the connection between motion and force, the equation-of-motion approach with its deterministic outcome continued and expanded well into the 19th century with more advanced formulations by Lagrange, Hamilton, and others. Development in other areas such as optics and electromagnetism did nothing to diminish the paradigm of determinism. The synthesis of all knowledge of electricity and magnetism into Maxwell's formulation of only four equations was a crowning achievement, not only solving to some extent the deficiency of action-at-a-distance required by Newton's laws, but also reinforcing the causality-through-determinism view. Given the sources of electromagnetism, namely charges and currents, one could calculate not only the static electromagnetic field but also the components of electromagnetic waves which develop in a dynamic situation.

The assumption implicit in this work was that nature followed patterns which were orderly and reproducible. Cause and effect were clearly operative even if the agency of communication was not always known. The point was, that if sufficient knowledge as to the precise form of nature's forces were available then the trajectories and velocities of all particles could be defined, at least in principle.

Physical determinism was so successful and seemingly all-pervasive that other areas of knowledge could not ignore its presence. The clockwork picture of the physical universe suggested by Newtonian thought seemed to threaten the formulation of that philosophical position which held that man had free will. After all, if molecular motion is determined by equations of motion and if man's brain is composed of molecules, wherein lies his freedom if Newton's laws prevail? Even the biological scientists made a tentative nod to the terminology of physics by describing the phenomenon of life as due to a 'vital force' Further afield one senses a kind of inexorable determinism. In literature, the many coincidences in the novels of Charles Dickens hardly seem to be chance occurrences and the sure tracing of minutiae by Sherlock Holmes precisely determine the solutions of mysteries in the works of A. Conan Doyle.

2. *Probabilistic models*

Yet physical science was not quite the uncracked monolith it appeared to be. Certain other directions and problems had begun to develop before the beginning of the 20th century. During the early part of the 19th century a somewhat more pragmatic science was developing to handle problems associated with interactions of energy, heat, and work. The science of thermodynamics, which eventually described relationships between macroscopic properties of bulk substance, such as heat, temperature, pressure, volume and so forth, did not even attempt the equation of motion approach endemic to mechanics and electromagnetism. The thermodynamicists seemed content with less than total knowledge and, in fact, the second law of thermodynamics is rampant with uncertainty: "No cyclic process exists whose sole effect is to extract heat from a substance and convert it entirely to work."[2]

Thermodynamics might have settled into the domain of applied physics and chemistry without much attention from pure science if there had not been theoreticians interested in establishing a microscopic basis for the macroscopic thermodynamics laws. It is at this juncture where the first break with the deterministic Newtonian tradition occurs. The reason for this departure can be seen from a consideration of the magnitude of the challenge to theoretical study by Newtonian methods.

Given a bulk amount of material, perhaps a mole of gas, there are about 6×10^{23} molecules, each of which has three position coordinates and three velocity or momentum coordinates, and each molecule interacts, however weakly, with all the other molecules. The description of this configuration by Newton's equations of motion would result in over 10^{24} coupled differential equations whose solutions would be virtually unattainable even with present day computer techniques. In the latter part of the 19th century such a system

[2] Colin J. Thompson, *Mathematical Statistical Mechanics,* Macmillan, New York, 1972, p. 40.

of equations was patently intractable. Clearly some alternative method of analysis was required.

It turned out that the mathematics of probability, with its genesis in such dubious applications as games of chance, was the instrument for development of a microscopic basis for thermodynamics. This new type of modeling, called statistical mechanics, was developed by James Clerk Maxwell, Ludwig Boltzmann, Willard Gibbs and others toward the end of the 19th century. The equation of motion approach was abandoned and in its place probability distributions, f(p,r), of momenta and position coordinates were calculated. These distributions were then used to determine statistical averages over all molecules. Such averages were found to correlate very well with the bulk properties of thermodynamics.

Even after the new mechanics for microscopic objects—quantum mechanics—came into being, statistical mechanics, suitably modified, was still used to make the connection between large quantities of microsystems and bulk matter behavior. This modified version is referred to as quantum statistical mechanics and still has its *raison d'etre* in the ability to deal with many-particle systems.

$$\frac{-\hbar^2}{2m} \nabla^2 \psi + V\psi = \frac{i\hbar \partial \psi}{\partial t}$$

The success of statistical mechanics undoubtedly influenced the further growth of probability models. Today, one of the most widely used probability models is that of the stochastic process, a probability model which develops in time. Stochastic models are applied in many areas including the physical sciences, engineering, economics and the social sciences. We think of nuclear decay processes, lineups (queues) at service stations, reliability studies, models of computers, random motions of air pollutants, to name just a few specific applications. All of these models have in common a willingness to sacrifice detailed knowledge of individual components of the system in exchange for good estimates of average or bulk behavior. The whole question of whether the system under consideration really is probabilistic is deferred. For example, persons in a queue like to think that their actions and those of their neighbors are non-random and, in fact, purposeful. This may be the case, but as long as the population *as a whole* and *in this context* behaves with a certain amount of randomness under uniform conditions, the system may be modelled by probability.

Let us conclude this section by noting the strength and wide applicability of the probabilistic model. But we are aware that despite the success of such models, they do not seriously challenge the notion of inherent determinism (from one cause or another) in matter. The appearance, from the model, is that the system is governed by chance, but is that really the case in the deepest parts of nature? We now look at a brand of physics which more closely addresses the issue of 'chance-in-principle.'

Quantum Mechanics—A Quick Look

At the beginning of the 20th century the view of nature as Newtonian in detail and statistical in bulk was receiving serious challenge. The difficulties arose from experimental work which attempted to approach the microscopic domain of atoms, electrons and other, as yet unseen, objects. Whereas the problems of statistical mechanics were of mathematical intractability, the attempt now was to discover the actual nature of these microscopic objects. A whole array of phenomena, including discrete spectral lines, black-body radiation, the photoelectric effect, and low temperature specific heats, to name the more famous cases, forced a substantial revision in the basic notions of deterministic mechanics. Ultimately this revision of mechanics would come to be known as quantum mechanics and appear initially in the complementary but equivalent formulations of wave mechanics by Erwin Schrodinger and matrix mechanics of Werner Heisenberg.

Probably the first thing to note is that quantum mechanics is, self-consciously, a theory of measurement. It does not concern itself with the might-be situation. Those who developed the theory worked very hard to make a theory which would explain and predict 'observables' only. The reasons for this bias lie with the unusual experimental findings which sometimes suggested one picture of reality and sometimes another.

What are the pictures that physicists use? In fact there have only been two pictures for the scientist, the particle and the wave. These pictures are so fundamental as to be almost undefined or primitive terms. They obey certain kinds of equations and have certain physical properties. If we were to look at all the equations of classical (pre-quantum) physics we could assign each equation either to a particle picture or a wave picture. (However, there is a kind of mixing as the appropriate wavelength tends toward zero.) In quantum physics, matter seems to exhibit both pictures, although the particular picture which is emphasized depends upon the situation.

Let us discuss just one example which illustrates much of what we are alluding to. Students of elementary physics are familiar with the wave phenomenon of interference. A beam of light is directed through a pair of closely spaced slits. On a screen behind the slits a series of intensity maxima and minima (fringes) are observed. The explanation is based upon the individual wavelets reinforcing or cancelling each other, and the positions of the fringes are determined by a well known mathematical function of the wavelength, the slit width, and slit separation. In other words, the wave theory of light predicts the interference effect very well.

A similar experiment can be done with particles, such as electrons. The electrons are streamed at the two slits and register on the screen via a scintillation mechanism. The set of spots on the screen produces discernible maximum and minimum densities of spots—the familiar interference pattern. In every respect the beam of electrons behaves like a wavefront. One might even surmise that the electron is, in some sense, wavelike. In fact one can associate with the electron's momentum a certain wavelength:

$$\lambda = \frac{h}{p}$$

where h = Planck's constant, p is the momentum, and λ is the wavelength.

Now comes the mystery—in both the electron and light experiments. If the electron stream density is made very low then only one electron goes through the double slit at a time giving one spot each on the screen, and there is no way of precisely calculating where that spot will be. The probability of the electron hitting a given region is only proportional to the intensity of spots when many electrons bombard the slits. Similarly, when an extremely low intensity light beam is used, the same phenomenon occurs. It is possible to turn down the intensity until there is a high probability that only one photon ('particle' of light) travels through the slit configuration at a time. Again the scintillations on the screen gradually accumulate until an interference pattern identical to the high intensity pattern is observed. Once again there has been interference even though, in this case, the photons travel pretty much by themselves. For the purposes of this discussion, the important point is the probabilistic behavior of both the electrons and photons. One cannot predict the slit through which individual photons and electrons will travel. How can one predict the precise location of a scintillation on the screen? Only a probability (or probability density) may be calculated for a given position on the screen. And this is the fundamental change from classical physics which one finds in microscopic systems. This change results in the physical theory called quantum mechanics.

Probably the 'chance' or 'indeterminancy' aspect of quantum physics is most visible in the uncertainty relation of Heisenberg. This relation exists between pairs of complementary dynamical quantities such as position and momentum, or angular momentum and phase. In terms of probability density functions for a given experiment one could say that the spread, Δx, in a space-probability function is never less than Planck's constant. That is, if the positions of particles in an ensemble are very well known then there is a fairly broad range of momenta available for the particles, and vice versa. One cannot know (or measure even ideally) the value of both quantities simultaneously to an arbitrary degree of accuracy.

Another form of the uncertainty relation holds between the variables of energy and time. One common example is radioactive decay. A radioactive nucleus is in a state whose energy is not precisely defined and therefore there is a certain width, ΔE, to its energy distribution. The nucleus decays quickly or slowly but with an average decay time, Δt. Note that the time of individual decays cannot be predicted exactly. Once again the product, $\Delta E \cdot \Delta t$ is greater than Plancks constant alluded to in the previous paragraph. The decay process is then another example of the probability nature of quantum physics.

The probability density function itself is based upon a mathematical quantity called the wave function, usually denoted as Δ. This function has a specific form for a given physical situation which is determined by the solution of a complicated equation (Schrodinger's equation):

$$\frac{-\hbar^2}{2m} \nabla^2 \psi + V \psi = \frac{i\hbar \partial \psi}{\partial t}$$

Without going into the details, we note that V is a function which describes the forces involved, and therefore Schrodinger's equation plays a role for the ψ function analogous to that of Newton's equation for the path of a particle. (Note that ψ is not a physical observable like the elements of Newton's equation of motion; ψ has even been referred to as a tendency or effort similar to Swedenborg's 'conatus.') Therefore the time development of the ψ function is quite determined by the forces and initial state of the system. In this sense quantum physics incorporates both deterministic and probabilistic features—causality and chance. As Max Born put the case in 1926: "The motion of particles conforms to the laws of probability but the probability itself is propagated in accordance with the laws of causality."[3]

Discussion

What then is the contribution of probability to our view of the world? For the use of probability to pre-quantum or non-quantum models the answer seems fairly straightforward. Probability is a pragmatic tool to be used because of lack of knowledge or intractable mathematics. Probability models predict averages which are very useful quantities, and without stochastic models we would certainly have less control of our environment and less efficient use of our resources.

Could one not ask if probability in quantum physics is not also the tool of the ignorant? There is a minority position in physics which suggests that there are 'hidden variables' which really control microscopic events, and that our inability to develop a theory incorporating such variables is only due to lack of ingenuity. However, the majority view is that quantum mechanics is a complete theory in itself and that the ψ function represents the maximum possible information on a system. In his book *Mathematical Foundations of Quantum Mechanics*[4], J. Von Neumann analyzed the logical completeness of quantum mechanics in order to see if hidden variables not subject to uncertainty might be incorporated in the model structure. He was able to show that such a process led to contradictions with the theory and thereby with valid predictions of experimental results. This problem was also attacked by other theoreticians from various perspectives, each time with the result that quantum mechanics

[3] Max Born, "Quantem Mechanik der Stobvorgänge," *Zeitschrift für Physik*, 38; 803, 1926.

[4] Von Neumann, A., *Mathematical Foundation of Quantum Mechanics*, Princeton University Press, Princeton, 1955.

must stand alone and that the uncertainty limitations are real.

Given the formidable predictive power of quantum mechanics and its durability in literally thousands of tests since the year of its invention, are we not faced with the problem of chance-in-reality for the material world?

Before examining this question further, consider the position taken in the theological writings of Swedenborg, who penned these works almost 200 years earlier than the advent of quantum theory. Swedenborg wrote at a time when only a very few mathematicians such as Bernoulli and de Moivre were concerned with the calculus of probability, and long before the classical treatise "Théorie Analytique des Probabilitiés" by La Place illustrated the use of probability beyond just games of chance. The idea of 'chance' or 'fortune' is really the central thrust of Swedenborg's statements, although one may infer a broader view of probability.

Chance or fortune are really just ultimations of Divine Providence: "...fortune...is ocular evidence that the Divine Providence is in the veriest singulars of man's thought and actions" (DP 212). And in *Arcana Coelestia* 6493: "They [angels] also confirmed the fact that there is no such thing as chance, and that apparent accident, or fortune, is Providence in the ultimate of order, in which all things are comparatively inconstant."

Clearly the Swedenborgian view is that chance and the validity of the probability view cannot be fully understood if seen only from the framework of the material world. Ultimately the spiritual cause of chance is not a kind of divine dice throwing as Einstein suggested the quantum mechanists believed. The Divine influence is purposeful and orderly according to the structure and operation of Providence, which maintains the twin purposes of leaving man in freedom while working for his salvation. For man is truly saved only if he is saved in freedom.

These considerations lead to the possibility that quantum mechanics does give the maximal representation of the material world. The question is not so much whether there is chance-in-principle in the deepest parts of nature, but rather whether the appearance of chance at the microscopic level is not a kind of epistemological barrier, Divinely provided, whereby influx from the spiritual world may remain hidden from the probing instruments of science. In fact, the metaphysical problem of chance-in-principle (for the material world) loses importance. At this level of nature, the question may not be, "What is?" but rather "What can we know?"