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V
FORMS: VARIETY WITHIN RECURRENCE

In this work we have emphasized the complementary natures of religion and science, especially as to their respective views of physical reality and their methods of description of that reality. We now focus on one important aspect of creation where the connection is quite novel and yet very suggestive of a simplicity that may underlie the complicated appearance that nature presents. This feature is the possible connection between the recurring forms at various levels of spiritual and natural reality, as described in the Writings, and mathematical sets called fractals, as generated numerically and naturally. We begin with a summary description of recurrence in creation as described in the Writings.

Recurrence and Duality

Recurrence and variety do not exist in a vacuum but are properties of the forms of reality. One of the most pervasive forms of reality is the dual, human form. In his theological Writings, Swedenborg emphasizes the dual structure of natural and spiritual reality. The main work on creation, *Angelic Wisdom Concerning the Divine Love and the Divine Wisdom*, establishes this framework as follows: "When there is *Esse* (being) there is *Existere* (taking form); one is not possible apart from the other. For *Esse* is by means of *Existere,* and not apart from it. This the rational mind comprehends . . . . And since one is possible with the other, and not apart from the other, it follows that they are one, but one distinctly." (DLW 14)

The pre-eminent example of duality is the Lord himself whose essence is Love and whose form is Wisdom. As a being in his Creator's image man is also a form of love and wisdom. This dual human form manifests itself on many levels. On the spiritual level, love and wisdom are paralleled by good and truth. These, in turn, are achievable through the dual human capacities of freedom and rationality associated with the human will and understanding. It is the orderly joining together of all these complementary pairs that promotes spiritual growth. On the natural level the human duality is evident in the symmetry of the physical body, in the complimentary roles of the heart and lungs, nucleic acids and proteins, and even in the double stranded structure of the DNA molecule itself. It is not very extravagant to suggest that this complementary duality must be characteristic of every human throughout the universe.

On a larger scale the human form is reflected in the structure and social makeup of the Spiritual world. The entire Heavens form what Swedenborg referred to as a Grand Man. Societies take on roles corresponding to bodily organs, and, when viewed from a distance, even have the same appearance as these organs. Within a given society this same general structure also occurs. Even earthly society, in an imperfect way, also acts as an organic body, with functions and mechanisms analogous to those of the human body.

Clearly, the human form is pervasive. Yet aside from the evident universality of the human form, two attendant characteristics stand out. These are a) the *repetition* of patterns at the different levels of nature, and b) the *variety* of repetitions consistent with the overall human form.

The student of biology sees abundant evidence of both features throughout nature—some obvious examples are, a cross-section of muscle tissue, the details of a tree leaf, and the complex but somewhat ordered geometric patterns of neuron aggregations. In each case increasing magnifications provides similar patterns. The microcosm reflects the macrocosm in an array of apparently endless variations on a theme.[1]

In a previous discussion of discrete levels, we noted many examples, from the molecular level to the subatomic level, of repeated structures in physical science. Recurrence properties are also evident in the motions and energies of these microscopic systems. As the particles are examined more deeply, new but similar physical systems are unfolded. While these similarities are striking and suggestive of recurrence, modern mathematics now provides a unifying paradigm. Mathematics has suggested a model whereby geometric structures can explicitly show repeatability

[1] E. J. Brock has discussed Swedenborg's philosophic thoughts on form, both in their historical context and in comparison with the structure of crystals and protein fibers. See *New Philosophy* 85 (1982): 84-95.

and variety. While this model is a pale imitation of nature's reality it can provide insight into that reality and contribute to the religion-science dialogue.

Mathematical Similarity and Variety

The mathematical model we refer to is the *Fractal,* a term which encompasses many possible geometric structures. Fractals are not easily defined. In his book *The Geometry of Fractals,* Benoit Mandelbrot, a founder of the subject, suggests a first example drawn from nature rather than mathematics: the coastline of an island. The coastline has many unusual properties. Consider the problem of measuring the coastline length. If one uses a fairly coarse measuring device the length is readily measured and found to be finite. But as the measuring device becomes capable of discerning smaller distances and curvatures, the coastline length is found to be longer; and as the measuring device becomes infinitely precise the coastline is found to be infinitely long! Yet the contained land mass is finite. Furthermore, the coastline also has the property of having similar appearances over long and short sections of its length, an example of repeated patterns on different scales.

A strictly mathematical example of a fractal is shown in figure 1. While this figure has a realistic appearance of a tree branch, in fact, the picture is computer generated with a program containing only a few lines of code![2] Note the similarity of pattern at different magnifications—repeated patterns. In mathematics this property is called *self-similarity.*

But for purposes of analysis let us begin with a very simple fractal known, for historical reasons, as the *Cantor* Set. The Cantor set is formed by iterated operations on the points of the real number line. Start with the line extending from zero to one. Remove the middle third, leaving two pieces at either end. Then remove the middle third of each of the two pieces thereby leaving four pieces. Repeat the process an infinite number of times as indicated in figure 2. The remaining points are the Cantor set.

The Cantor set has some unusual properties. First, the generating process, a process with an infinite number of steps, seems unusual. Second, the set has an infinite number of points but a length of zero.

[2] The parameters for this particular diagram are due to W. D. Withers, as given in his unpublished notes "Introduction to Fractals" (1988).

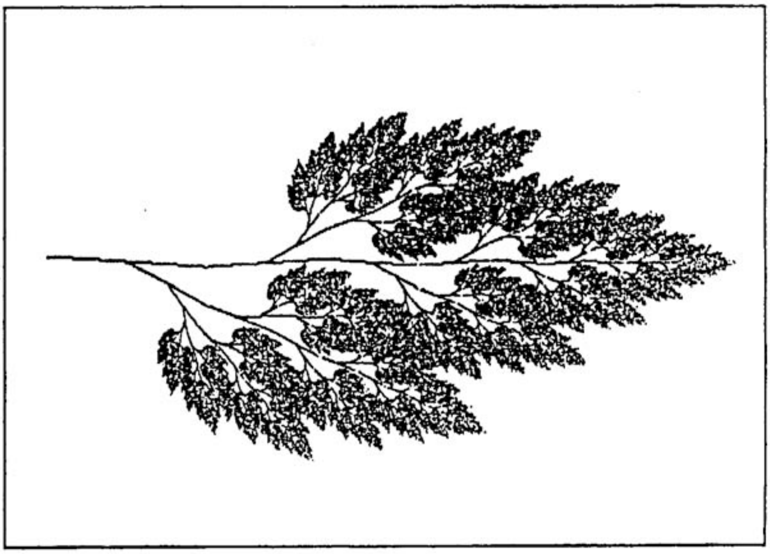

Fig. 1. Computer generated picture of a branch. (The transformation probabilities are variable. See below for explanation.)

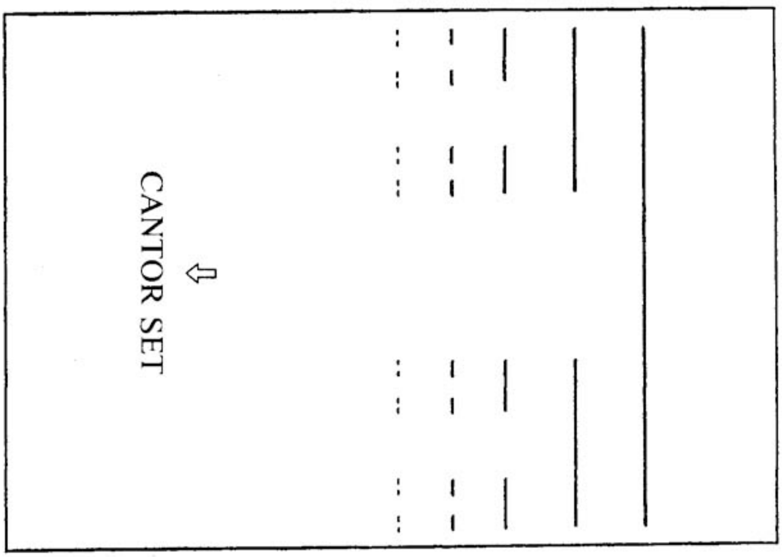

Fig. 2. The Cantor set as the limit set of an infinite number of iterations of the process of removing "the middle third".

Third, it can be shown that the dimension of the set is *fractional,* lying between the dimension of one for a finite line segment and of zero for a finite collection of points. (Its dimension is actually:

$$\log 2/\log 3 \approx 0.6309298...).$$

The Cantor set seems to exist in some sort of limbo state between points and lines.

The unusual properties of the Cantor set, especially its fractional dimension, are hallmarks of fractal sets. For our purposes the Cantor set exhibits the self-similarity property (repeated patterns) very well. By observing different bits of the set in figure 2 it is clear that the set has the same appearance at different magnifications—or levels.

Fractal sets can be generated by computer algorithms. The 2-dimensional Cantor set in figure 3 is good example. Like the basic Cantor set this figure shows the properties of self-similarity, at least to within the resolution of the computer pixels. Increasingly, smaller portions of the diagram have the same pattern as larger portions. While the technicalities of this diagram are outside the scope of this discussion, it will be useful to know that the algorithm consists of 4 mathematical transformations. Similarly, the algorithm for the leaf (or branch) of figure 1 is generated from 3 transformations.

The 2-dimensional Cantor set of figure 3 clearly exhibits the self-similar property. But fractals can also model the second characteristic variability. The 2-dimensional Cantor set shown in figure 4 differs from that in figure 3 in that there is shading or variability in the density of points of the set. In each set of 4 blocks, whether on a small or large scale, the density is highest in the lower left corner and least in the upper right corner of the block. In nature this variability gives the interest and beauty that we marvel at. The computer leaf of figure 1 shows this variability, whereas, by contrast, the computer leaf of figure 5 does not. In fact the latter is less interesting and less complete.

The difference between the variable and non-variable fractals lies in the relative frequencies of the transformations used in their generation. In figure 5 the three mathematical transformations are all repeated equally often, whereas the more interesting leaf of figure 1 uses the transformations with relative frequencies 2, 30, and 25. Each transformation is chosen randomly with these characteristic statistical weights. Therefore

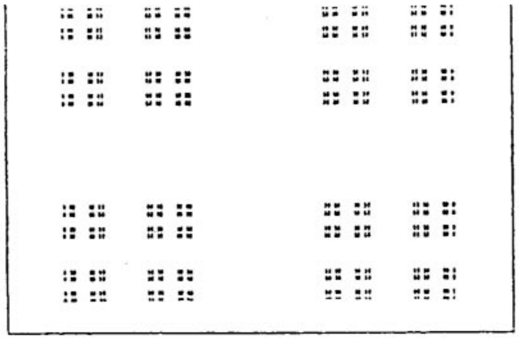

Fig. 3. A 2-dimensional Cantor set generated by the computer using an iterated function system. The probabilities for each transformation are equal.

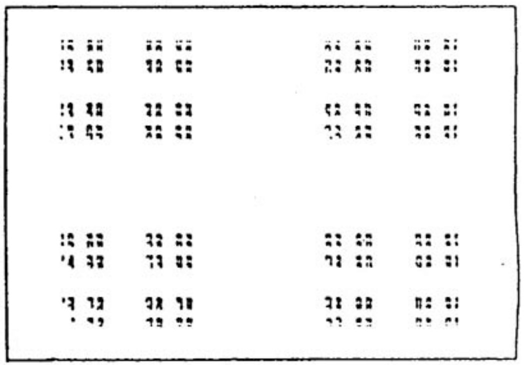

Fig. 4. A 2-dimensional Cantor set generated by the computer. The probabilities for the transformations are unequal. Note the resultant shading, although the effect is somewhat obscured by the resolution of the diagram.

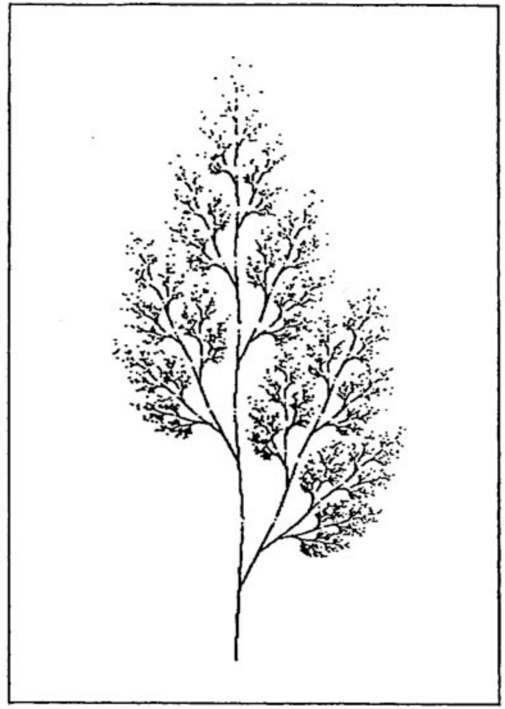

Fig. 5. Computer generated picture of a tree branch. The transformation probabilities are equal and therefore the picture lacks the variety found in figure 1.

the particular variation in the fractal (leaf) is probabilistic. (We note that the comparison between the variable and non-variable fractal is based upon equal total numbers of transformation iterations say, 200,000 iterations.)

Two other computer replicas of natural objects are given in figures 6 and 7, clouds, and a spiral galaxy. These diagrams illustrate both self-similarity and variability, and are generated by the same algorithm (different transformations) as those in figures 1 and 4. The second characteristic we seek, variability within the general pattern, is made possible by using unequal probabilities for the transformations.

Having demonstrated the ability of a computer to artificially generate self-similar variable structures, we now ask if such structures are found to have a greater reality based in nature. We have already alluded to the example of the coastline. Certainly leaves, landscapes, and snowflakes all have the fractal appearance. Yet it is instructive to examine, in a little more detail, some of the phenomena which scientific studies have shown to exhibit fractal properties.

The Science of Self-Similar Variety

Much of the strength and pervasiveness of modern science comes from its ability to define and solve small, manageable problems. These small problems are then used as models to approximate aspects of the behavior of nature. In the final step these partial pictures are additively combined to synthesize a more complete view of nature. The difficulty with this reductionist method is that many of the complexities and subtleties of nature tend to be ignored as objects of serious study.

But nature is manifestly complex, and in the last three decades, the scientific study of complexity has become more fashionable. In this section we describe some scientific areas of study where complex behavior and self-similar variability come together.

The symmetric geometric structure of well-formed crystals is an elementary and well-known example of order and symmetry in nature. However, the time dependent growth behavior of crystals is a much newer area of study. In this recent work the spatial patterns of the crystal dendrites are seen to have complex structure. The fractal-like pattern of

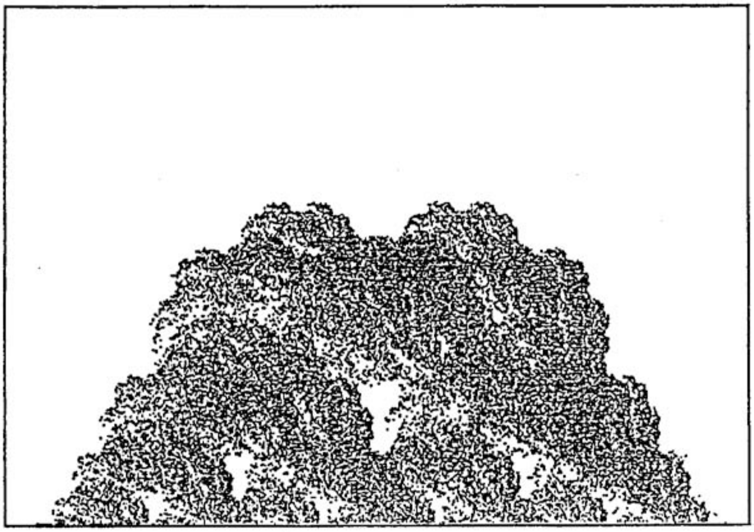

Fig. 6. Computer generated picture of clouds. The transformation probabilities are unequal.

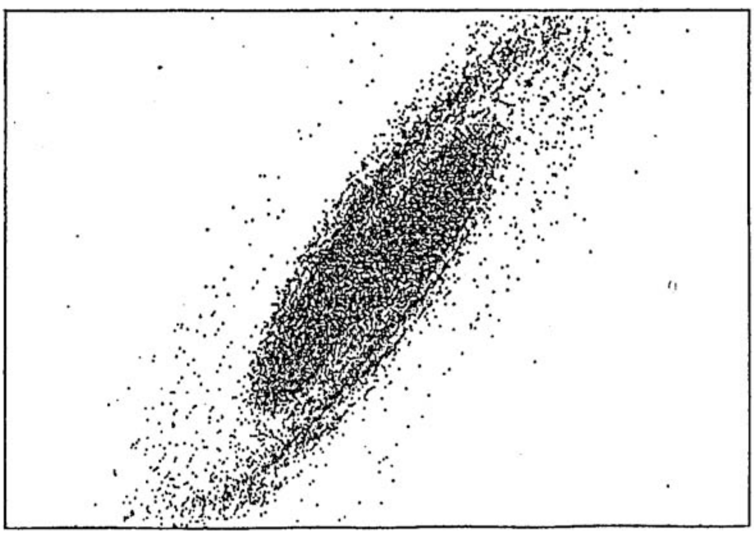

Fig. 7. Computer generated picture of a spiral galaxy. The transformation probabilities are unequal.

growth for a crystal of ammonium bromide is shown in figure 8.[3] The figure shows the dendrite 2-dimensional outline at 10 second intervals. The contours appear to exhibit both self-similarity and variety.

Another example of an evident fractal structure is the human brain. The convolutions of cerebral surface is reminiscent of the coastline fractal. Dimension calculations of this surface have been made from magnetic resonance images taken at various orientations. In the normal human brain the cerebral surface bounded by the gray matter has a dimension of 2.6. The technique of magnetic resonance imaging is particularly sensitive to changes in proportions of gray and white matter, and therefore the fractal dimension may prove to be a useful device in assessing pathological conditions.[4]

The dendritic behavior of growing crystals and the convolutions of the cerebral surface exemplify self-similarity and variety in static forms. But these properties are also evident in the *dynamics* of nature, although in a more subtle way. We now describe some examples of this level of self-similar variety.

Dynamical Systems

We can define a dynamical system, somewhat abstractly, as a set equations that characterize the time dependent behavior of certain quantities. These quantities might include the concentrations in a chemical reaction, the charge, current, and voltage in an electrical circuit, or the angular velocity and displacement of a driven pendulum. In these examples the particular time dependent behavior may be modeled by kinetic theory, Ohm's law, and Newton's second law, respectively. But whatever the particular example or governing law, the study of dynamical systems attempts to find the time dependent behavior of the given quantities, known generically as dynamical variables.

Until fairly recently dynamical systems were solved by mathematical analysis, *in those cases where analytic solution was possible.* Where analytic solutions were not found the system, or at least that version of

[3] From A. Dougherty and J. P. Gollub, *Phys. Rev. A* 38 (1988): 3043 by permission of the author, J. P. Gollub.

[4] S. Majumdar and R. R. Prasad, *Computers in Physics*, 2 (1988) 6: 69.

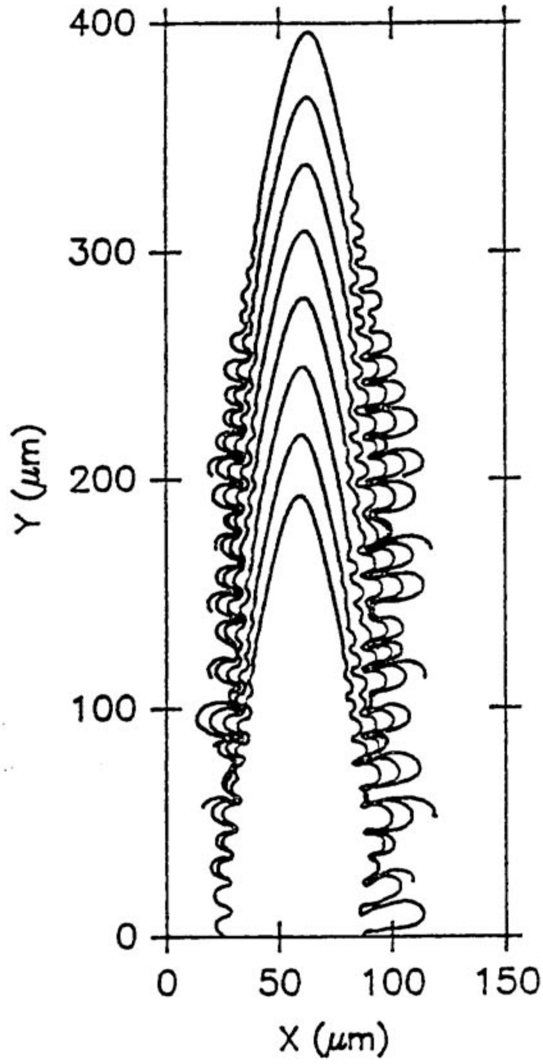

Fig. 8. Spatial contours of a growing dendrite of a crystal of NH_4Br. The contours are taken at 10 second intervals. From A. Dougherty & J. P. Gollub, *Phys. Rev. A,* 38 (1988): 3043; by permission of the author, J. P. Gollub.

the system, was often relegated to the backwaters of science. Systems that did provide analytic solutions usually exhibited stable, regular (often periodic) motion. A very large class of systems which could be counted on to behave in a regular fashion, usually periodically, were *linear* systems. These linear systems provided a raison d'etre for the concentration of collegiate scientific education on linear algebra, linear differential equations, and linear systems analysis.

Dynamical systems can have unstable motions when the dynamical variables are coupled together in a *nonlinear* fashion. This circumstance allows for a delicate and complex interplay of motions. As we have suggested such systems are difficult to analyze. Early attempts by the French mathematician, Henri Poincaré, to analyze nonlinear systems form the basis for current analytic studies.[5] Yet the primary incentive for the study of these complex systems is the advent of digital computers. These machines can provide numerical solutions where analytic solutions fail to exist, and this numerical work can, in turn, promote the development of new mathematics and new insight into the nature of complexity.

The new mathematics of nonlinear unstable systems is the mathematics of fractals, and the discovery of further fractal properties is often derived from the study of these systems. Contrary to earlier thinking such systems are ubiquitous and their behavior is now classified as *chaotic.* Chaotic behavior and fractal structures are now observed across the entire spectrum of natural phenomenon. For example, chaotic dynamics are found in oscillating chemical reactions, turbulent fluid flow, laser dynamics, and driven mechanical systems. These systems exhibit chaos in conditions which are very different from equilibrium states, and often under circumstances where there are forces acting to counter the "natural" state of the system. These conditions might be compared to the interaction of two causal chains. It is their complex interplay that leads to chaotic behavior.

An important consequence of the instability of chaotic systems is their *sensitivity to initial conditions.* That is, two identical dynamical systems whose motions are initiated in slightly different ways, will very quickly diverge in behavior. For all practical purposes the long term motion of a chaotic system is statistical.

[5] H. Poincaré, *Les Méthodes nouvelles de la Méeanique Céleste* (Paris: Gauthier-Villars, 1892; reprint, New York: Dover Pub. Inc., 1957).

Let us make these ideas more concrete with two examples. In the first example we discuss the physical configuration and kinetic equations associated with a chaotic chemical reaction. In the second example we develop a more detailed picture of a mechanical system, the driven pendulum. This description will include evidence for the fractal nature of the pendulum motion.

a) **Chemical Reactions**

The usual model of chemical kinetics (the time development of reactant concentrations) provides a simple introduction to the notion of a dynamical system. Let us consider the very simple chemical reaction:

$$A \underset{k_r}{\overset{k_f}{\rightleftarrows}} B$$

In this reaction, reactant A becomes B at a rate proportional to the concentration of A, and B becomes A at a rate proportional to the concentration of B present. The proportionality constants for each process are k_f and k_r, respectively. If N_A and N_B are the concentrations of A and B then their rates of change in time t are given by the equations:

$$\frac{d\mathbf{N}_A}{dt} = -\mathbf{k}_f\mathbf{N}_A + \mathbf{k}_r\mathbf{N}_B$$

$$\frac{d\mathbf{N}_B}{dt} = \mathbf{k}_f\mathbf{N}_A - \mathbf{k}_r\mathbf{N}_B$$

This pair of differential equations constitutes a dynamical system. The concentrations, $\mathbf{N}_A$ and $\mathbf{N}_B$ are the system dynamical variables. The system is linear in its dependence on concentrations and couplings between them. Analytic solution of the equations provides a time dependent decay of $\mathbf{N}_f$ and $\mathbf{N}_r$ toward their equilibrium values. Linear systems with more variables will still have regular behavior, usually of a periodic nature. While our school laboratory experience may not have included periodic or oscillating reactions, they are common to living systems. One such reaction is the much studied Belousov-Zhabotinskii reaction. However the **B−Z** reaction is also nonlinear and therefore is prone to instability—chaotic behavior. The relevant features of the B-Z reaction may be represented by a simpler reaction:

$$A + B \underset{k_r}{\overset{k_f}{\rightleftarrows}} C$$

The existence of two reactants on the left side provides for nonlinear coupling in the corresponding dynamical system:

$$\frac{d\mathbf{N}_A}{dt} = -k_f \mathbf{N}_A \mathbf{N}_B + k_r \mathbf{N}_c$$

$$\frac{d\mathbf{N}_B}{dt} = -k_f \mathbf{N}_A \mathbf{N}_B + k_r \mathbf{N}_c$$

$$\frac{d\mathbf{N}_c}{dt} = +k_f \mathbf{N}_A \mathbf{N}_B - k_r \mathbf{N}_c$$

This system can be rendered unstable by a continual infusion of **A**, **B,** and **C** at some appropriate rate (with some modification of the equations), thereby preventing the system from decaying to an equilibrium state. Under such conditions the **B-Z** reaction exhibits the many characteristics of chaotic behavior.

This chemical example allows us to suggest that the prerequisites of chaotic motion are a) non-linear coupling between the dynamical variables, b) continued pressure to keep the system from reaching a stable steady state, and, as it turns out, c) at least three dynamical variables. With these features in mind, we now examine a simple mechanical system, the driven pendulum. Sufficient detail will be presented to demonstrate the fractal aspects of its chaotic motion.

b) **The Driven Pendulum**

Pendula have long been the object of scientific enquiry—at least since Galileo's observations of swaying lamps in the cathedral at Pisa. The motion of the driven pendulum is governed by Newton's second law—mass times acceleration equals the sum of the applied forces. For the driven pendulum the forces are a) the frictional damping of the motion, b) the restoring force of gravity, and c) a periodic driving or pushing force. Mathematical manipulation of Newton's law provides a form which is manifestly that of a dynamical system.

$$\frac{d\omega}{dt} = -(1/q)\omega - \sin\theta + g\mathrm{cso}\varphi$$

$$\frac{d\theta}{dt} = \omega$$

$$\frac{d\varphi}{dt} = \omega_D$$

The presence of the nonlinear couping terms and the driving force, $g\cos\varphi$, are necessary for chaotic behavior. The three dynamical variables are θ, the angular displacement of pendulum, ω, the angular velocity, and φ, a variable proportional to time. Solutions to this system are periodic (stable) or chaotic (unstable) depending on the values of three parameters, q a damping factor, g the amplitude of the driving force, and w_D the angular frequency of the periodic driving force. Each parameter plays a different role. Increased damping (decreased q) tends to stabilize the motion, whereas increased forcing (increased g) tends to destabilize the motion. Values of the drive frequency, ω_D, which differ from the natural frequency of one, provide a destabilizing competition between the force of gravity and the driving force. The overall effect is that the pendulum is capable of exhibiting a variety of periodic and chaotic motions.

The fractal nature of chaotic pendulum motion is evident in certain geometric constructions involving the dynamical variables, $(\theta, \omega, \varphi)$. An understanding of these constructions will require a little background knowledge.

We begin with a description of *phase space.* The phase space of a dynamical system of N variables is an N-dimensional Euclidean space, having each axis correspond to one of the variables. For the pendulum the axes are labeled as θ, ω, and φ (or time). The real space motion (as we see it!) is represented in phase space by a sequence of points moving positively in the φ (time) direction with the θ and ω taking their instantaneous values in the plane perpendicular to the φ axis.

For example, if the motion in real space is a periodic, back and forth motion (of the type associated with a child being pushed on a swing), then, when the angular displacement θ is greatest, the angular velocity ω is zero. Similarly, when the pendulum is at the bottom of its motion, θ is zero and ω has its largest magnitude. Therefore the motion in phase space is a spiral directed positively along the time axis as shown in figure 9. (The time coordinate only needs to have a length equal to the period of the drive cycle because the drive mechanism is periodic. In technical terms, φ is assigned periodic boundary conditions.)

An important aspect of this diagram is that it also represents the *attractor* of the motion: the set of points to which the phase-space motion is eventually drawn. The spiral is the phase-space motion after the initial transient behavior has died away. Whatever the initial trajectory of the pendulum, its phase space motion is "attracted" to the spiral. Therefore the spiral is the *attractor.*

Chaotic states can also be represented in phase space. But the real space pendulum motion is now unstable, varying continuously in no perceivable pattern. Even after initial transients have died away the motion is still very complex and always changing. Therefore the phase space attractor is correspondingly complex, with an apparently infinite number of foldings. A motion picture simulation of the attractor would show a continual stretching and folding of the attractor as nearby phase points diverge from each other. A chaotic attractor exhibiting this complex structure is shown in figure 10.

Because chaotic attractors are so complex, it is sometimes convenient to use only a 2-dimensional cross section of the attractor, cut across the time or φ axis. The section would be a plane with θ and ω as the coordinate axes. This device, called a *Poincaré* section, is especially useful when the system is driven periodically since it always has the same

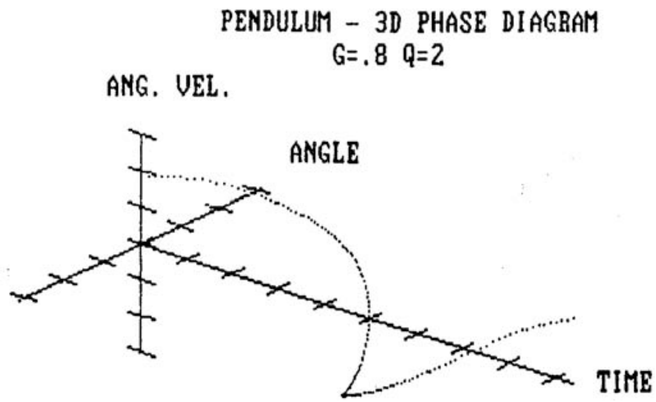

Fig. 9. A 3-dimensional phase space diagram for a lightly driven pendulum. The pendulum motion is periodic, similar to the motion of a swing in a playground. Just as the motion of the playground swing always repeats itself so does the motion of the spiral trajectory in phase space. The spiral is called an attractor.

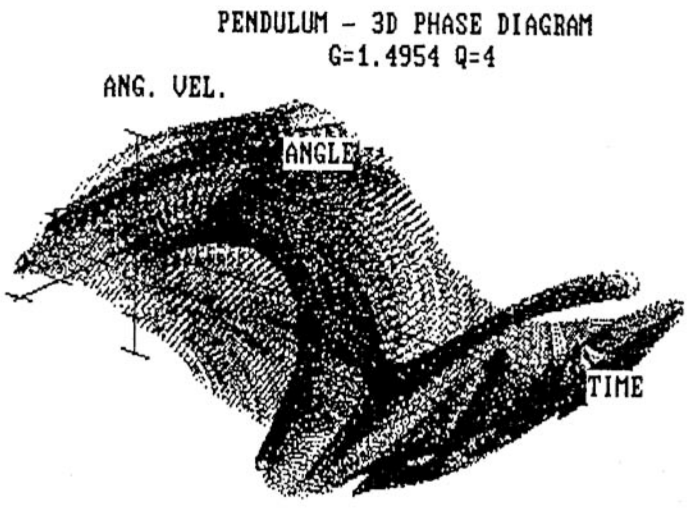

Fig. 10. A 3-dimensional phase portrait for a strongly driven chaotic pendulum. The pendulum motion is non-periodic. It never repeats itself. The configuration is called a strange attractor.

appearance at a particular point in the drive cycle. (If the pendulum motion is like that of a playground swing, then the section would consist of a single point—the (θ, ω) point which is repeated at that time in the cycle.) Figure 11 shows the Poincaré section taken at the beginning of each cycle for the chaotic pendulum attractor shown in figure 10. Although still complex, this figure highlights important properties of the chaotic state.

Perhaps the most striking feature of the chaotic Poincaré attractor is that, at any point on the attractor, there is a local direction along which the attractor seems to flow quasi-continuously and there is a corresponding perpendicular direction along which the attractor has a *fractal* appearance. One indication of fractal geometry is self-similarity, and the magnification shown in figure 12 of part of the attractor seems to suggest this property. (Note that this diagram is limited by the resolution of the computer screen and the finite number of points generated by the numerical simulation.) In fact, damped chaotic systems do have fractal attraetors and for this reason these attractors are called *strange attractors.*

Besides the self-similarity property the strange attractor also exhibits a fascinating structural variety. Any given piece of the attractor has the common fractal appearance, but the details vary endlessly throughout the attractor.

As a further check on the fractal nature of the attractor one can calculate (with the computer) the fractal dimension of the Poincaré attractor, and of the full attractor embedded in the 3-dimensional space. In this particular state the Poincaré section has a dimension of 1.2 and the full attractor has a dimension of 2.2. As expected, the dimensions are fractional.

Beside illustrating the fractal property of geometric sets associated with the chaotic pendulum, the dimension calculation provides a truly amazing connection between the fractal geometry and the physics of the pendulum. It turns out that the fractal dimension is quantitatively related to the frictional damping factor q, of the pendulum. Increased damping constricts the bends and folds of the attractor. Therefore the fractal dimensions of the Poincaré sections and attractors will vary with changes in pendulum damping. Fractal characteristics are intimately related to the dynamics that produces them.

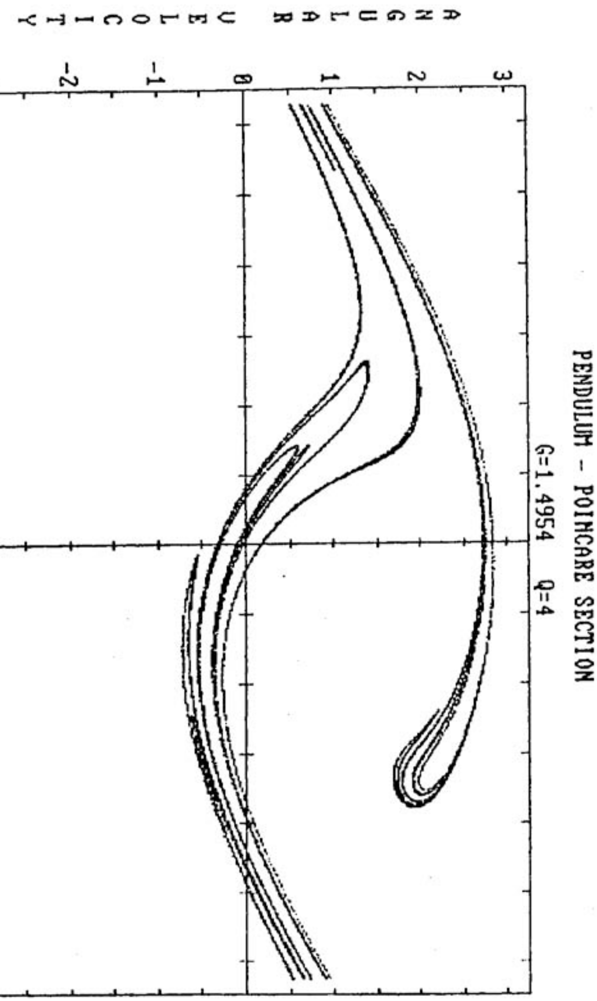

Fig. 11. A Poincaré section of the strange attractor for the chaotic pendulum. The picture is less complicated than the phase portrait but still shows the stretching and folding phase motion.

The attractor fractal dimension is also related to the apparent statistical behavior of chaotic dynamical systems. We recall that chaotic systems show a sensitive dependence on initial conditions. In reality these conditions can never be precisely determined, and therefore the uncertainty of the state of a chaotic system grows in time. There is a progressive loss of information about the system. The rate of loss is a property of the dynamics, which, in turn produces the fractal strange attractor. Therefore the rate of loss of information depends on the attractor fractal dimension. Furthermore this loss of memory in chaotic systems goes to the heart of the nature of irreversible processes. Clearly these features all suggest deep connections between geometry and dynamics.

The chaotic pendulum can illustrate fractal behavior in ways that are beyond even those described here. But this brief survey is perhaps sufficient to suggest the intimate connection between variable self-similarity and chaotic dynamical systems.

Naturally Occurring Chaos

The set of mathematical equations which comprise the dynamical system provide a relatively direct means to analyze periodic and chaotic motion. However, in the real world of nature, such equations are often not readily available. Even approximate mathematical models can be complex and their validity may be limited to artificial experimental arrangements. Nevertheless experimental work in chaos is an expanding field.

A common experimental procedure is to measure the time variation of some variable of a system in the laboratory and then analyze its mathematical properties. Such analysis can include reconstruction of the system phase portrait, spectral analysis, and calculation of fractal dimension. The results of these studies are then compared to the behavior of dynamical systems. Let us briefly mention some of the more exotic areas of study.

There is much interest in the chaotic aspects of physiology. Studies have been initiated into the nature of periodic and chaotic chemical reactions in the human body. As a processor of continued infusions of energy, the body can be thought of as chemical reactor operating far from

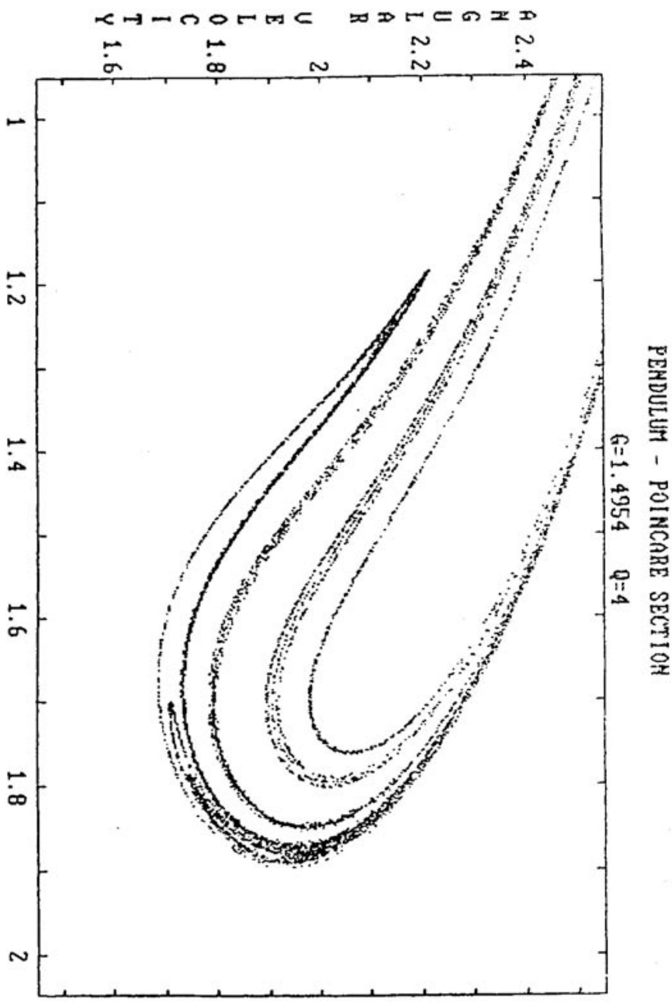

Fig. 12. Magnification of part of the Poincaré section showing the self-similar fractal structure of the cross section. In the local perpendicular direction the flow appears to be smooth.

equilibrium conditions. Complex dynamics has been observed in cellular metabolism, some of which may correlate with pathological conditions.[6] Various physiological conditions including tremors, dyskinesias, and epilepsy have been studied. For example, initial studies suggest that the epileptic seizure may be a physiological corrective mechanism for moving from a chaotic state to a periodic state. Work has also been done on cardiac fibrillation as a chaotic system. (It is worth noting that medical science has long characterized certain phenomenon as "chaotic," in the sense of randomness, but the newer usage has a more precise and complex meaning.)

Various population studies have been made. Animal population studies show the reality of chaotic behavior. Similarly the spread of disease exhibits both periodicity and chaos. For example the study of the epidemiology of childhood diseases—measles, mumps, and chicken pox—showed an interesting contrast. The work of W. M. Schafferand M. Kot[7] focused on New York and Baltimore during the period before the measles vaccine. Their analysis showed that the incidence of mumps and chicken pox were periodic phenomena whereas the measles data supported chaotic patterns and the existence of a strange attractor.

An endless recitation of chaos studies could be given. The scope of this work is such that one prominent investigator entitled a paper, "Is the Solar System Stable?"[8] Yet the sample cited is perhaps sufficient to suggest both the diversity and reality of chaotic behavior. That this phenomenon is intimately connected to mathematical fractals, the mechanism of self-similar variation, is a truly marvelous thing. Perhaps the sense of wonder stems from the both the many levels of connectedness amongst these things, and the developing completeness of this large part of our world view. Yet this connectedness should not be surprising since

[6] See P. E. Rapp in Arun Holden (ed.) *Chaos* (Princeton: Princeton Univ. Press, 1986), pp. 179-208.

[7] In Arun Holden (ed.) *Chaos{*Princeton: Princeton Univ. Press, 1986), pp. 158-178.

[8] J. K. Moser, *Mathematical Intelligencer* 1 (1978): 65-71.

revelation has long emphasized the integrity and harmony of creation. Perhaps this is a partial fulfillment of the promise described in *True Christian Religion:*

> Here [in the New Church] one is permitted to enter with the understanding into all its interior truths, and also to confirm them by the Word. The reason is that its doctrines are truths in series from the Lord, revealed by means of the Word; and *confirmation of these by rational considerations* opens the higher reaches of the understanding and so elevates it into the light which the angels of heaven enjoy. This light in its essence is truth, and in this light the acknowledgment of the Lord as the God of heaven and earth shines in all its glory. (TCR 508:5. Italics added.)■